1°

$$\dot{\underline{x}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 2 & 1 & -2 \end{bmatrix} \underline{x} + \begin{bmatrix} 6a & 12b \\ 0 & 0 \\ 1 & -6c \end{bmatrix} \underline{u}$$

$$\underline{y} = \begin{bmatrix} 9 & 1 & 4 \\ 0 & 0 & 1 \\ 0 & 3 & -1 \end{bmatrix} \underline{x}$$

(a) $\lambda_1 = -2 \qquad \lambda_2 = -1 \qquad \lambda_3 = 1$

$$\underline{e}_1 = \begin{bmatrix} 1 \\ -2 \\ 4 \end{bmatrix} \qquad \underline{e}_2 = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} \qquad \underline{e}_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$E = \begin{bmatrix} 1 & 1 & 1 \\ -2 & -1 & 1 \\ 4 & 1 & 1 \end{bmatrix} \qquad e^{\Lambda t} = \begin{bmatrix} e^{-2t} & 0 & 0 \\ 0 & e^{-t} & 0 \\ 0 & 0 & e^{t} \end{bmatrix}$$

(b) $\underline{\Phi} = \underline{E}\, e^{\Lambda t}\, \underline{E}^{-1}$

$$\begin{bmatrix} \frac{e^{t}}{3} + e^{-t} - \frac{e^{-2t}}{3} & \frac{e^{t} - e^{-t}}{2} & \frac{e^{t}}{6} - \frac{e^{-t}}{2} + \frac{e^{-2t}}{3} \\ \frac{e^{t}}{3} - e^{-t} + \frac{2}{3}e^{-3t} & \frac{e^{t} + e^{-t}}{2} & \frac{e^{t}}{6}\ \frac{e^{-t}}{2} - \frac{2e^{-2t}}{3} \\ \frac{e^{t}}{3} + \frac{e^{-t}}{1} - \frac{4}{3}e^{-2t} & \frac{e^{t} - e^{-t}}{2} & \frac{e^{t}}{6} - \frac{e^{-t}}{2} \overset{+}{-} \frac{\cancel{4}e^{-2t}}{3} \end{bmatrix}$$

(c) $E\dot{\underline{z}} = AE\underline{z} + B\underline{u}$

$\dot{\underline{z}} = \Lambda \underline{z} + E^{-1}B\underline{u}$

$$B = \begin{bmatrix} 6a & 12b \\ 0 & 0 \\ 1 & -6c \end{bmatrix}$$

$$E^{-1}B = \begin{bmatrix} \frac{1}{3}-2a & -2c-4b \\ 6a-\frac{1}{2} & 3c+12b \\ 2a+\frac{1}{6} & 4b-c \end{bmatrix} \begin{matrix} \leftarrow \\ \leftarrow \\ \leftarrow \end{matrix}$$

$$\left.\begin{matrix} \begin{bmatrix} \frac{1}{3}-2a & -2c-4b \end{bmatrix} \equiv [0, 0] \\ \text{"or"} \\ \begin{bmatrix} 6a-\frac{1}{2} & 3c+12b \end{bmatrix} \equiv [0, 0] \\ \text{"or"} \\ \begin{bmatrix} 2a+\frac{1}{6} & 4b-c \end{bmatrix} \equiv [0, 0] \end{matrix}\right\}$$ then not controllable

(d) $\underline{y} = C\underline{E}$

$$C\underline{E} = \begin{bmatrix} g+14 & g+3 & g+5 \\ 4 & 1 & 1 \\ -10 & -4 & 2 \end{bmatrix}$$

$$\begin{matrix} \Downarrow & \Downarrow & \Downarrow \\ \neq 0 & \neq 0 & \neq 0 \end{matrix}$$

obs

(2)

$$PI = \int_0^1 \left[x^2 + 2u^2\right] dt$$

$$\dot{x} = 6x + u$$

(a)

$$H = \left[x^2 + 2u^2\right] + \lambda\left[6x + u\right]$$

$$\frac{\partial H}{\partial u} = 0 = 4u + \lambda \Rightarrow \boxed{u^{\circ} = -\frac{\lambda}{4}}$$

$$H^{\circ} = x^2 + \frac{\lambda^2}{16}2 + \cancel{\lambda} - \lambda\left[6x - \frac{\lambda}{4}\right]$$

(b) $\dot{x} = 6x - \lambda/4$

$$\dot{\lambda} = -2x - 6\lambda \Leftarrow \frac{\partial \lambda}{\partial t} = -\frac{\partial H^{\circ}}{\partial x}$$

(c)

$$\begin{bmatrix} \dot{x} \\ \dot{\lambda} \end{bmatrix} = \begin{bmatrix} 6 & -1/4 \\ -2 & -6 \end{bmatrix} \begin{bmatrix} x \\ \lambda \end{bmatrix}$$

③ $U \rightarrow \boxed{} \rightarrow Y$

$$\frac{Y}{U} = \frac{s+3}{s^3+8s^2+19s+2}$$

(a) $$\dot{\underline{x}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2 & -19 & -8 \end{bmatrix} \underline{x} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 3 & 1 & 0 \end{bmatrix} \underline{x}$$

(b) $$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2 & -19 & -8 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \qquad C = \begin{bmatrix} 3 & 1 & 0 \end{bmatrix}$$