Routh Stability

Special Case

first column zero

ie $\quad s^3 + 2s^2 + s + 2 = 0$ $\Leftarrow$ Char eqn

$$\begin{array}{c|cc} s^3 & 1 & 1 \\ s^2 & 2 & 2 \\ s^1 & \cancel{\frac{(2)(1)-(2)(1)}{2}} \to 0 & \\ s^0 & & \end{array} \quad \Rightarrow \quad \begin{array}{c|cc} s^3 & 1 & 1 \\ s^2 & 2 & 2 \\ s^1 & \epsilon & \\ s^0 & \cancel{\frac{(\epsilon)(2)-(2)(0)}{\epsilon}} \to 2 & \end{array}$$

$$\begin{array}{c|cc} s^3 & 1 & 1 \\ s^2 & 2 & 2 \\ s^1 & 0 & \\ s^0 & 2 & \end{array}$$

s^2 row $\Rightarrow$ aux polynomial $2s^2+2 = P(s)$

s^1 row $\Leftarrow$ $j\omega$-axis pole(s)

$P(s) = 2s^2+2$

$P(s) = 0 = 2s^2+2 \Rightarrow s^2+1=0$

$$\boxed{s = \pm j}$$

row of zeros

consider $s^5+2s^4+24s^3+48s^2-25s-50=0$

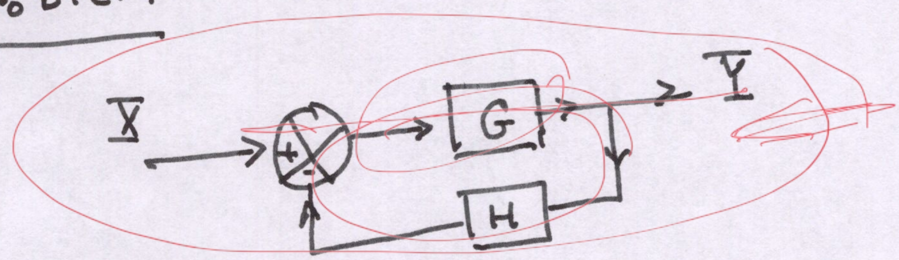
s^5	1	24	-25
s^4	2	48	-50 ← aux poly $P(s)$
s^3	$\frac{(2)(24)-(1)(48)}{2} \to 0$	$\frac{(2)(-25)-(-50)(1)}{2} \to 0$	

$$P(s) = 2s^4+48s^2-50$$

$$\boxed{\frac{dP(s)}{ds} = 8s^3+96s}$$

s^5	1	24	-25
s^4	2	48	-50
s^3	8	96	
s^2	$\frac{(8)(48)-(2)96}{8} \to 24$	$\frac{8(-50)-(2)(0)}{8} \to -50$	
s^1	$\frac{(24)(96)-(8)(-50)}{24} \to 2704/24$		
s^0	$\frac{(-50)(2704/24)-(24)(0)}{2704/24} \to -50$		

Problem



$$G = \frac{4s+k}{s^2}$$

$$H = \frac{1}{s+2}$$

$$\frac{Y}{X} = \frac{G}{1+GH}$$

(a) Find Y/X

$1+GH$

(b) Char eqn

(c) Range of k for stable behavior

(d) when is the system marginally stable and the freq of oscillation soln

(a) $\dfrac{Y}{X} = \dfrac{G}{1+GH}$

(b) $1+GH = P(s) = 1 + \left(\dfrac{4s+k}{s^2}\right)\dfrac{1}{(s+2)}$

$p(s) = 0 = s^2(s+2) + 4s + k$

$0 = s^3 + 2s^2 + 4s + K$

(c)

$$\begin{array}{l|ll} s^3 & 1 & 4 \\ s^2 & 2 & K \\ s & \dfrac{(2)(4)-k}{2} & \\ s^0 & K & \end{array}$$

$\Longrightarrow$ stable: $8-k > 0 \Rightarrow 8 > k$

$\Longrightarrow$ $k > 0$

$$\boxed{8 > k > 0}$$

(d) need zero is 1st column

$$\begin{array}{l|ll} & 1 & 4 \\ s^2 & 2 & k \\ & 0 & \end{array}$$

$\Longrightarrow p(s) = 2s^2 + k = 0$

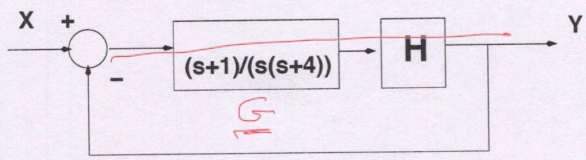
$\Longrightarrow k = 8$

$2s^2 + 8 = 0 \Rightarrow \boxed{s = \pm j2}$

$$\boxed{f = \frac{2}{2\pi}\ hz}$$

Name________________

1. Consider the unity feedback system given by the block diagram below.



a. If $E(s) = X - Y$ evaluate the transfer function $\frac{E}{X}$.

b. Determine $E(s)$ given $x(t) = u(t)$.

c. Determine $H(s)$ such that $e(\infty) = 0.10$.

(a) $\boxed{\frac{Y}{X} = \frac{GH}{1-(-GH)} = \frac{GH}{1+GH}}$

(b) $E = X - Y$

$\frac{E}{X} = 1 - \frac{Y}{X} = 1 - \frac{GH}{1+GH} = \frac{1+\cancel{GH}-\cancel{GH}}{1+GH}$

$\boxed{\frac{E}{X} = \frac{1}{1+GH}}$

$x(t) = u(t)$

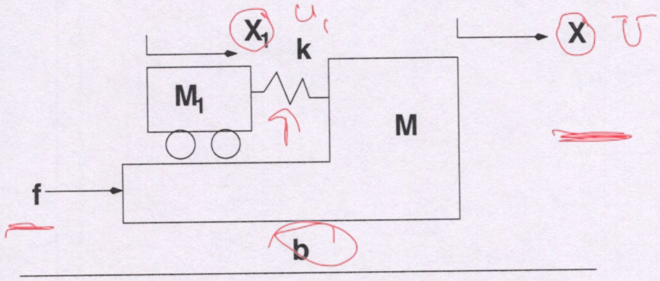
$X(s) \overset{?}{=} \frac{1}{s}$

$E(s) = \left(\frac{1}{s}\right)\left(\frac{1}{1+GH}\right)$

$e(\infty) = \lim_{s \to 0} sE(s) = 0.10$

5

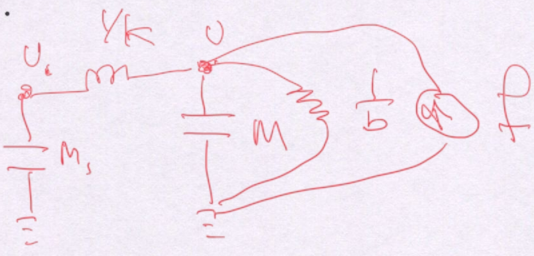
2. Consider the mechanical system where $f(t)$ is the force applied to mass M. The position x and x_1 denote the position of M_1 and M respectively. The variable k is the stiffness of a spring and b is the damper.(note $u = dx/dt$)



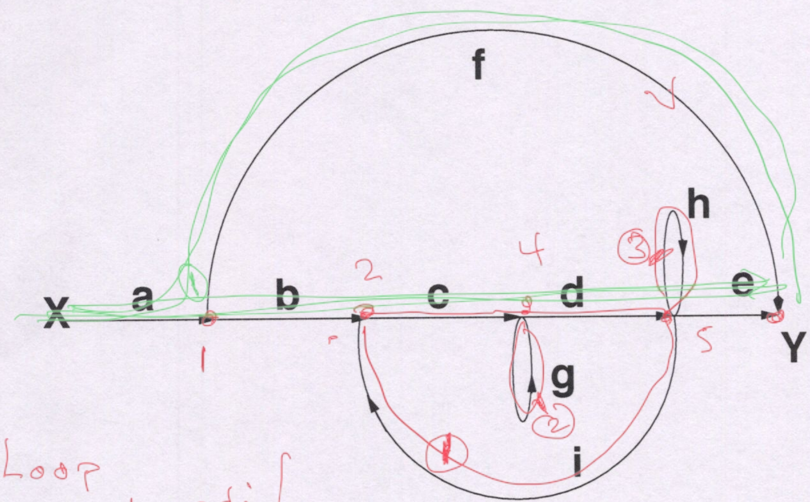
a. Using the mobility analogy, where force is the through variable and the velocity is the across variable, draw the electromechanical circuit.

b. Determine the governing equations of motion in ODE form.

c. Evaluate the transfer function of the Laplace transforms of the velocities $U_1(s)/U(s)$.



3. Given the signal flow, determine the transfer function $\frac{Y}{X}$ using Mason's Gain Formula.



Loop

245 $L_1 = cdi$

4 $L_2 = g$

5 $L_3 = h$

Forward $P_1 = abcde$

$P_2 = af$

$\Delta = 1 - [L_1 + L_2 + L_3] + [L_2 L_3]$

$\Delta_1 = 1 - 0$ Δ_2

4. Consider the unity negative feedback system where the characteristic equation $GH + 1 = 0$.

$$GH = \frac{K(s-4)}{(s+1)(s+2)(s+3)}$$

a. Determine the range of values of K where the system is stable.

.8

1.

(a) $$\frac{E}{X} = \frac{1}{1 + H\,\frac{(s+1)}{s(s+4)}}$$

(b) $$E = \left(\frac{1}{s}\right)\frac{1}{\left[1 + H\,\frac{(s+1)}{s(s+4)}\right]}$$

(c) $$\lim_{s\to 0} sE = e(\infty) = \frac{1}{\lim_{s\to 0}\left(\frac{H}{s}\,\frac{(1)}{4} + 1\right)} \Rightarrow 0.10$$

$$\boxed{H = ks}$$

$$\frac{1}{1 + k\frac{(1)}{4}} = \frac{1}{10} \qquad \boxed{k = \cancel{40}}\ 36$$

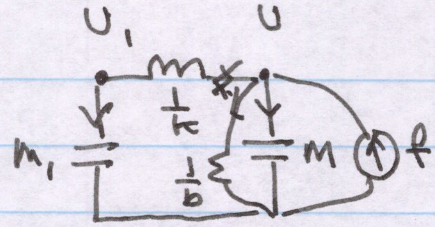
(d) $$1 + H\,\frac{(s+1)}{s(s+4)} \Rightarrow 1 + \frac{k(s+1)}{s+4}$$

$$(s+4) + k(s+1) = 0$$

$$(k+1)s + (k+4) = 0$$ stable

2.

(a)



(b)

node U: $$F = \frac{U}{\frac{1}{Ms}} + \frac{U}{\frac{1}{b}} + \frac{U - U_1}{\frac{s}{k}}$$

node U_1:

$$\frac{U - U_1}{\frac{s}{k}} = \frac{U_1}{\frac{1}{sM_1}}$$

$$\dot{f} = \ddot{u} M + b\dot{u} + (u - u_1)k$$

$$(\dot{u} - \dot{u}_1)k = m_1(\ddot{u}_1)$$

(c) $$\left(\frac{U - U_1}{s}\right) \cdot k = sM_1 U_1$$

$$(U - U_1)k = s^2 M_1 U_1$$

$$kU = U_1\left[s^2 M_1 + k\right] \Rightarrow \frac{U_1}{U} = \frac{k}{s^2 M_1 + k}$$

3.

$P_1 = abcde$

$P_2 = af$

$L_1 = cdi$

$L_2 = g$

$L_3 = h$

$\Delta = 1 - (L_1 + L_2 + L_3) + (L_2 L_3)$

$\Delta_1 = 1 - (0)$

$\Delta_2 = 1 - (L_1 + L_2 + L_3) + (L_2 L_3)$

$$\frac{Y}{X} = \frac{P_1 \Delta_1}{\Delta} + \frac{P_2 \Delta_2}{\Delta}$$

4

$$GH = \frac{k(s-4)}{(s+1)(s+2)(s+3)}$$

$1+GH=0 \quad (s+1)(s+2)(s+3)+k(s-4)=0$

$$s^3+6s^2+(k+11)s+(6-4k)=0$$

s^3	1	$k+11$
s^2	6	$6-4k$
s^1	$\frac{6(k+11)-(6-4k)}{6}$	$\Longrightarrow \frac{10k+60}{6}$
s^0	$6-4k$	

$$\frac{6k+66-6+4k}{6} = \frac{60+10k}{6} > 0 \quad \boxed{k>-6}$$

$$6-4k>0 \Rightarrow \boxed{\frac{6}{4}>k}$$

$$\boxed{k<\frac{6}{4}}$$