Linear stability

$$\boxed{\frac{\bar{Y}}{\bar{X}} = \frac{N(s)}{D(s)}}$$

Causal system

zeros $N(s) = 0$

poles $D(s) = 0$

Poles are the roots of $D(s) = 0$

$$D(s) = (s - s_1)(s - s_2) \ldots\ldots (s - s_N)$$

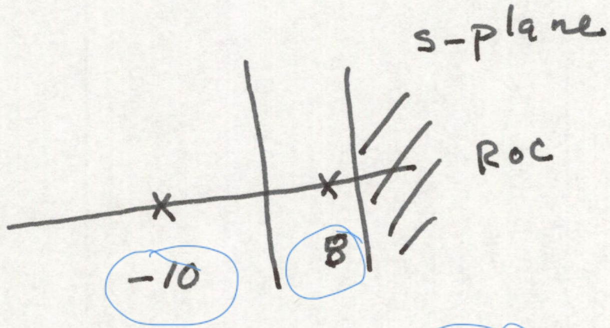
$$D(s) = s^{N} + a_1 s^{N-1} \ldots + \alpha s^{0}$$

real coeff

if the coeff are real

roots are $\begin{cases} \text{real} \\ \text{or} \\ \text{complex conj pairs} \end{cases}$

Relationship between root/poles and time response



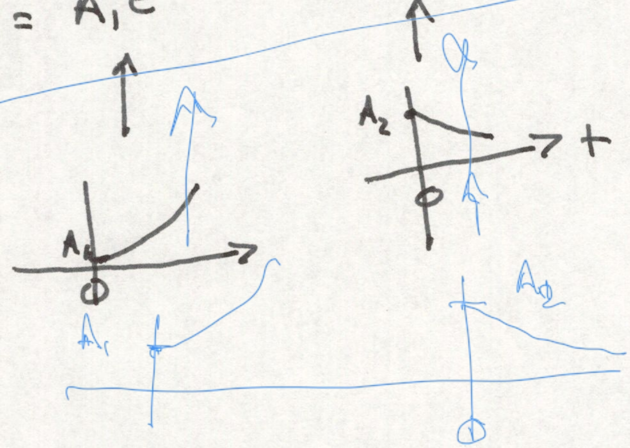
$$\frac{Y}{X} = \frac{N(s)}{(s+10)(s-8)} \qquad X = 1 \qquad x(t) = \delta(t)$$

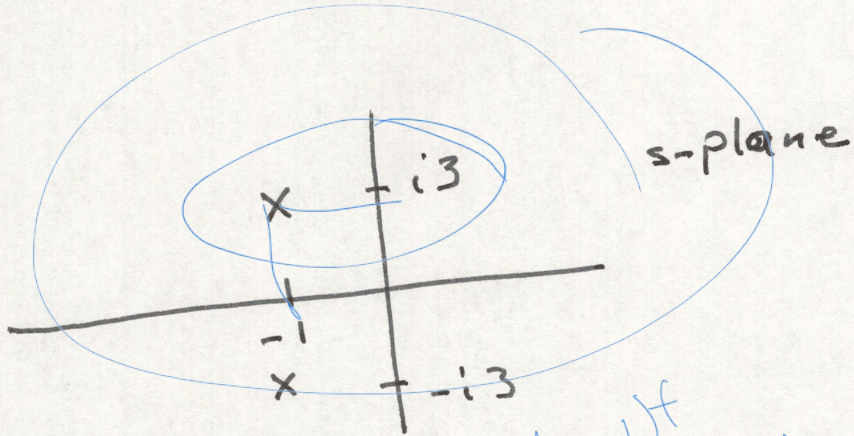
$$Y = \frac{N(s)}{(s+10)(s-8)}$$

$$y(t) = \left[\frac{N(s)}{s+10}\right]\Bigg|_{s=-8} e^{\cancel{5t}\,+8t} + \left[\frac{N(s)}{s-8}\right]\Bigg|_{s=-10} e^{-10t}$$

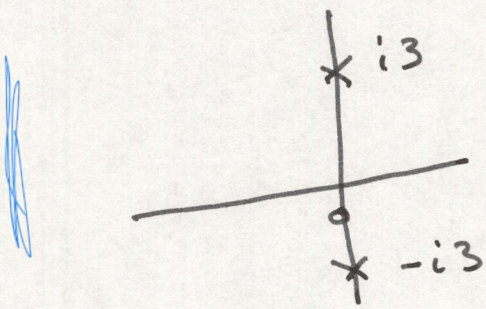
$t > t_0$

$$y(t) = A_1 e^{+8t} + A_2 e^{-10t}$$

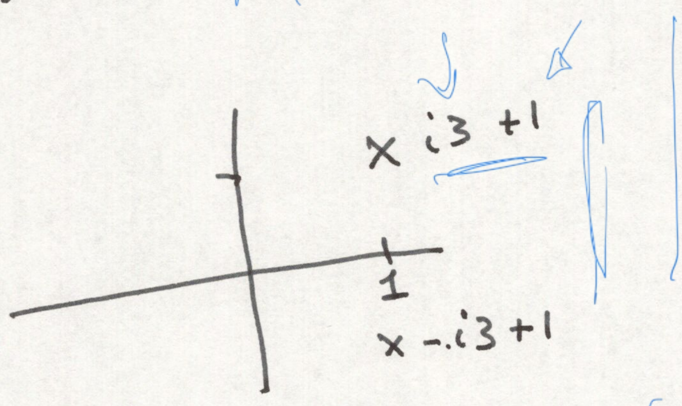




$y(t) = ? = A_1 e^{(i3-1)t} + A_2 e^{(-i3-1)t}$



$y(t) = ?\ A_1 e^{i3t} + A_2 e^{-i3t}$



$y(t) = ?\ A_1 e^{(i3+1)t} + A_2 e^{(-i3+1)t}$

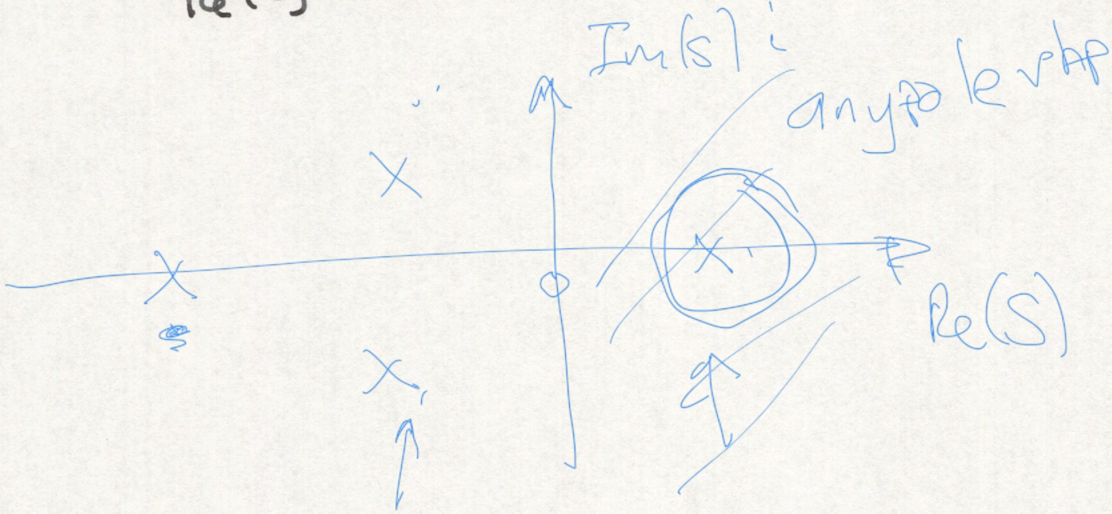
Stability

$$\boxed{D(s) = 0 \rightarrow s_1, s_2 \ldots s_N}$$

$Re(s_j) < 0$ stable

$Re(s_j) = 0$ marginally stable

$Re(s_j) > 0$ unstable



~~if~~ an

if all Poles have neg Re part the system is stable

Problem: Factorization

no analytical method to factor a general polynomial of degree higher than degree 3

Location / indication

Indication of poles in the lhp via Routh - method

$$D(s) = (s-1)(s+2)(s+3)$$

$$D(s) = s^3 + 4s^2 + s - 6$$

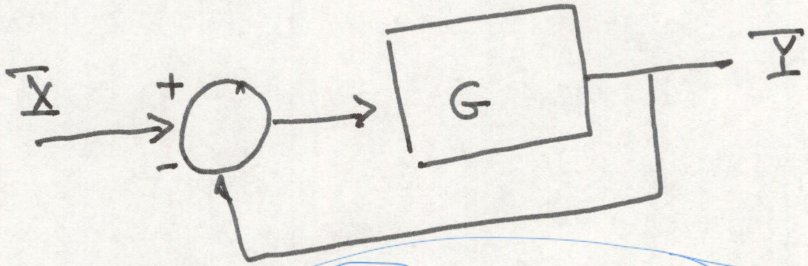
poles $s = 1$
$s = -2$
$s = -3$

Routh Table

s^3	1	1
s^2	4	-6
s^1	$\underbrace{\frac{4(1)-(1)(-6)}{4}}$ $\frac{10}{4}$	
s^0	$\frac{\frac{10}{4}(-6)}{\frac{10}{4}}$	

1 sign change

1 root in ~~lhp~~ rhp



$$G = \frac{k}{s(s^2+s+1)(s+2)}$$

$$\frac{Y}{X} = \frac{G}{\boxed{1+GH}} = D(s)$$

$\boxed{\text{Char eqn}}$ $\boxed{1+GH}$

$$1+GH = 0 \Rightarrow \boxed{s(s^2+s+1)(s+2) + k = 0}$$

Poles:

$$\boxed{s^4 + 3s^3 + 3s^2 + 2s + k = 0}$$

For what values of k is the system stable?

R-table

$$s^4 + 3s^3 + 3s^2 + 2s + k = 0$$

$$
\begin{array}{c|ccc}
s^4 & 1 & 3 & k \\
s^3 & 3 & 2 & \\
s^2 & \underbrace{\dfrac{(3)(3)-(1)(2)}{3}}_{\frac{7}{3}} & \underbrace{\dfrac{(3)(k)}{3}}_{k} & \\
s^1 & \underbrace{\dfrac{\left(\frac{7}{3}\right)(2)-k(3)}{7/3}}_{2-\frac{9}{7}k} & & \\
s^0 & \underbrace{\dfrac{\left(2-\frac{9}{7}k\right)k}{2-\frac{9}{7}k}}_{k} & &
\end{array}
$$

Stable

$$\Rightarrow 2 - \frac{9}{7}k > 0$$

$$\Rightarrow k > 0$$

$$2 - \frac{9}{7}k > 0 \Rightarrow \frac{14}{9} > k$$

$$k > 0$$

$$\boxed{\frac{14}{9} > k > 0}$$