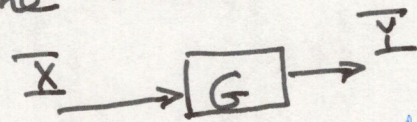


Steady State Error

Consider the Plant



cmd

reldt

the error

$$E = X(s) - Y(s)$$

$$E = X - GX$$

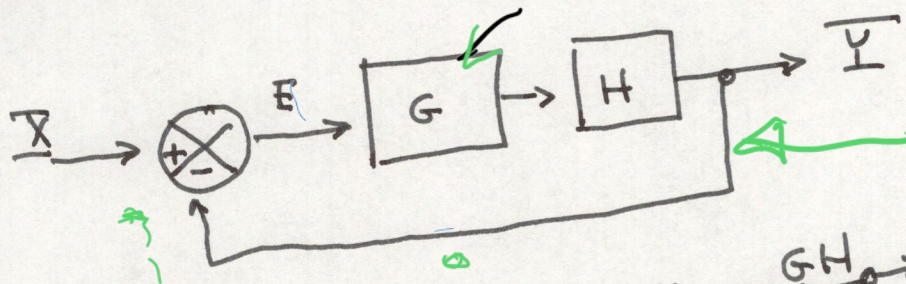
where

$$Y(s) = G(s)X$$

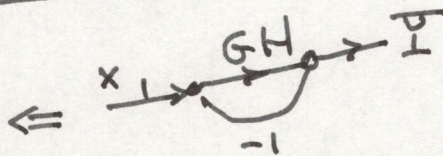
The error transfer function is

$$\frac{E}{X} = (1 - G)$$

Consider Unity ^{gain} neg. feedback system



$$1^{\circ} \quad \frac{Y}{X} = \frac{GH}{1+GH}$$



$$2^{\circ} \quad E = X - Y \Rightarrow \frac{E}{X} = 1 - \frac{Y}{X} = 1 - \frac{GH}{1+GH}$$

$$\boxed{\frac{E}{X} = \frac{1}{1+GH}}$$

Therefore

$$E(s) = \frac{\bar{X}}{1+GH}$$

The steady-state error
for a stable system

$$\rightarrow e(\infty) = \lim_{s \rightarrow 0} sE$$

Final value thm

Assumes char. eqn

$$1+GH = 0$$

yields
roots
in the left
half s-plane

$$E = \frac{X}{1+GH}$$

$$e(\infty) = \lim_{s \rightarrow 0} \left\{ \left(\frac{\bar{X}}{1+GH} \right) s \right\}$$

$$GH = \frac{N(s)}{s^n D(s)} \quad (\text{type } n) \leftarrow$$

n = to the number poles at the origin

and $\uparrow N(s) = \mathbb{C}_1$ ✓
 $\uparrow D(s) = \mathbb{C}_2$ ✓

$$1 + \frac{s+6}{s(s+4)} = 1 + GH$$

GH	type	$\uparrow GH$ s
$\frac{6}{s+4}$	0	$\frac{6}{4}$
$\frac{s+1}{s^2+3s}$	1	∞
$\frac{s+6}{s^3(s+1)}$	3	∞

$$s = -4$$

$$\frac{s+1}{s^2+3s} = \frac{s+1}{s(s+3)}$$

$$\Rightarrow \frac{s+1}{s(s+3)}$$

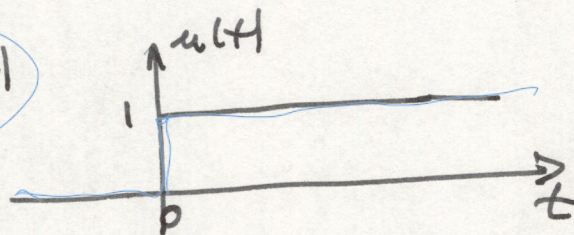
$$= \frac{1}{s(3)} = \frac{1}{3s}$$

$$\frac{s+6}{s^3(s+1)} = \frac{6}{s^3(1)} = \frac{6}{s^3} = \infty$$

Displacement error

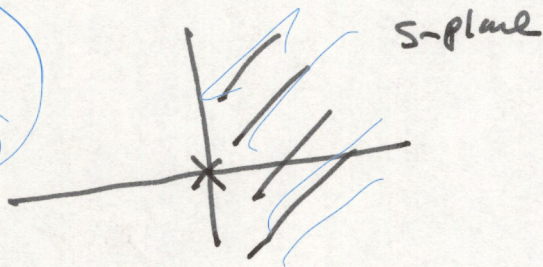
input

$$x(t) = u(t)$$



$$X(s) = \frac{1}{s}$$

$$ROC_x: \text{Re}(s) > 0$$



$$E(s) = \frac{X}{1+GH}$$

$$E(s) = \left(\frac{1}{s}\right) \left[\frac{1}{1+GH}\right]$$

$$\lim_{t \rightarrow \infty} e(t) = \lim_{s \rightarrow 0} s E(s) = \lim_{s \rightarrow 0} \frac{s}{s} \frac{1}{1+GH} = \lim_{s \rightarrow 0} \frac{1}{1+GH} = e(\infty)$$

$$\lim_{s \rightarrow 0} \frac{1}{1+GH} = e(\infty)$$

Type	$\lim_{s \rightarrow 0} GH$	$\lim_{s \rightarrow 0} \frac{1}{1+GH} = e(\infty)$
0	C	$\frac{1}{1+C}$
1	∞	0
2	∞	0

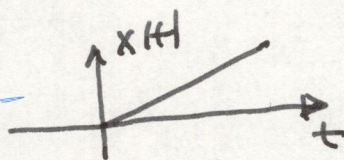
$\frac{GH}{1+GH}$
 $\frac{N(s)}{D(s)}$
 $\frac{C_1}{s} + \frac{C_2}{s^2} + \dots$

$$\frac{1}{s^2 + 2s + 4}$$

Velocity error

Input

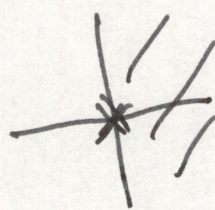
$$x(t) = tu(t)$$



$$\dot{x} = C$$

$$X(s) = \frac{1}{s^2}$$

$$ROC_X: \text{Re}(s) > 0$$



s-plane

$$E(s) = \frac{X}{1+GH}$$

$$E(s) = \left(\frac{1}{s^2}\right) \frac{1}{1+GH}$$

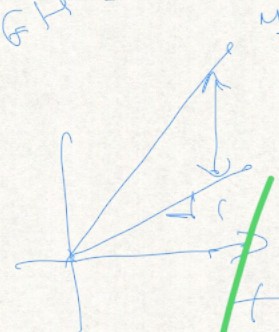
$$\int_t^\infty e(t) dt = \int_s^0 s E(s) ds = \int_s^0 \left(\frac{s}{s^2}\right) \frac{1}{1+GH} ds = \int_s^0 \frac{1}{s(1+GH)} ds$$

$$= \int_s^0 \frac{1}{s + sGH} ds$$

$$= \int_s^0 \frac{1}{s} \left(\frac{1}{1+GH} \right) ds = \int_s^0 \frac{1}{s(1+GH)} ds$$

$$e(\infty) = \int_s^0 \frac{1}{sGH} ds$$

Type	$\int_s^0 \frac{1}{sGH} ds$	$\int_s^0 \frac{1}{sGH} = e(\infty)$
0	0	∞
1	C	$1/C$
2	∞	0



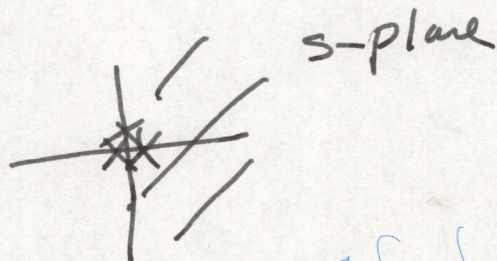
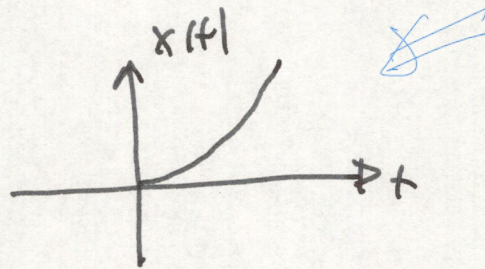
$$\int_s^0 \frac{1}{sGH} ds = \int_s^0 \frac{1}{s} \left(\frac{1}{GH} \right) ds = \int_s^0 \frac{1}{s} ds = \ln(s) \Big|_s^0 = \ln(0) - \ln(s) = -\infty - \ln(s) = \infty$$

Acceleration error

input $x(t) = \frac{t^2}{2} u(t)$

$\bar{X}(s) = \frac{1}{s^3}$

ROC_x: $\text{Re}(s) > 0$



$E(s) = \frac{\bar{X}}{1+GH}$

$E(s) = \left(\frac{1}{s^3} \right) \left(\frac{1}{1+GH} \right)$

$\int_0^\infty e(t) dt = \int_0^\infty \frac{1}{s^2 + s^2 GH} = \int_0^\infty \frac{1}{s^2 GH}$

$sE = \frac{s}{s^2(1+GH)}$

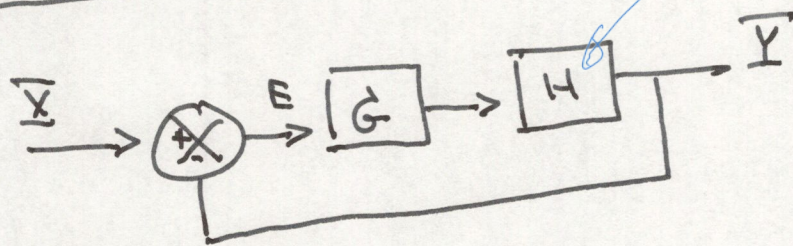
$= \frac{1}{s^2(1+GH)}$

$= \int_0^\infty \frac{1}{s^2 GH}$

$= \int_0^\infty \frac{1}{s^2 GH}$

Type	$\int_0^\infty \frac{1}{s^2 GH}$	$e(\infty) = \int_0^\infty \frac{1}{s^2 GH}$
$\Rightarrow 0$	∞	∞
$\Rightarrow 1$	∞	∞
$\Rightarrow 2$	∞	$1/4$
$\Rightarrow 3$	∞	0

Example



$$G = \frac{(s+1)}{(s+2)(s+3)}$$

Find $H(s)$ such that $e(\infty) = \frac{1}{10}$
for $x(t) = u(t)$

$$E(s) = X(s) \frac{1}{1+GH} = \left(\frac{1}{s}\right) \left(\frac{1}{1+GH}\right)$$

$$\lim_{s \rightarrow 0} s E = \lim_{s \rightarrow 0} e(t) = \lim_{s \rightarrow 0} \frac{1}{1+GH} = \lim_{s \rightarrow 0} \frac{1}{1 + \frac{(s+1)}{(s+2)(s+3)} H(s)}$$

$$= \frac{1}{10} = \lim_{s \rightarrow 0} \frac{1}{1 + \frac{1}{6} H(s)}$$

$$H(s) = K$$

prop controller

$$\frac{1}{10} = \frac{1}{1 + \frac{K}{6}} \Rightarrow \frac{K}{6} + 1 = 10$$

$$K = (9)(6)$$