

Mason's Gain Form,

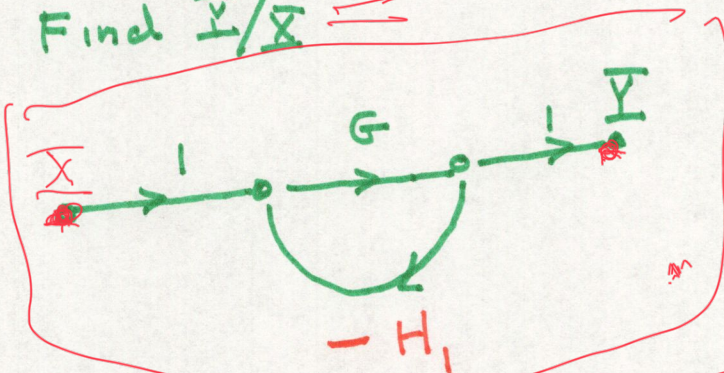
A. Build SFG

B. Find $\bar{Y}/\bar{X}$



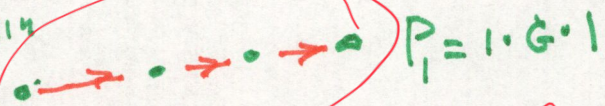
$$P_1 = G$$

A.



B.

Forward path gain



$$P_1 = 1 \cdot G \cdot 1$$

Loops



$$L_1 = G(-H) = -GH$$

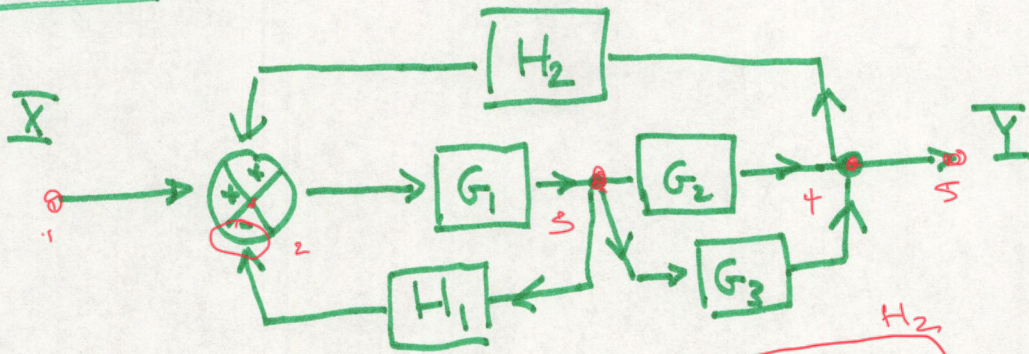
Mason's Gain Formula

$$\Delta = 1 - L_1$$

$$\Delta_1 = 1$$

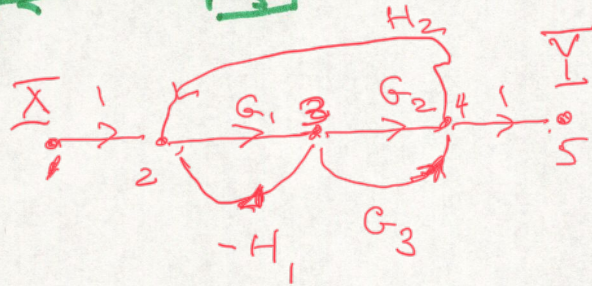
$$\frac{\bar{Y}}{\bar{X}} = \frac{P_1 \Delta_1}{\Delta} = \frac{G}{1 + GH}$$

Problem



A. SFG

B. Find Y/X



Forward

$\rightarrow \xrightarrow{1} \xrightarrow{G_1} \xrightarrow{G_2} \rightarrow P_1 = G_1 G_2$

$\rightarrow \rightarrow \rightarrow P_2 = G_1 G_3$ $\cancel{K_1}$ $\cancel{K_2}$ $\cancel{K_3}$
 $\boxed{\Delta_2 = 1}$

Loops

$L_1 = G(-H_1)$
 $L_2 = G_1 G_2 H_2$
 $L_3 = G_1 G_3 H_2$

$\Delta = 1 - (L_1 + L_2 + L_3) + (0)$

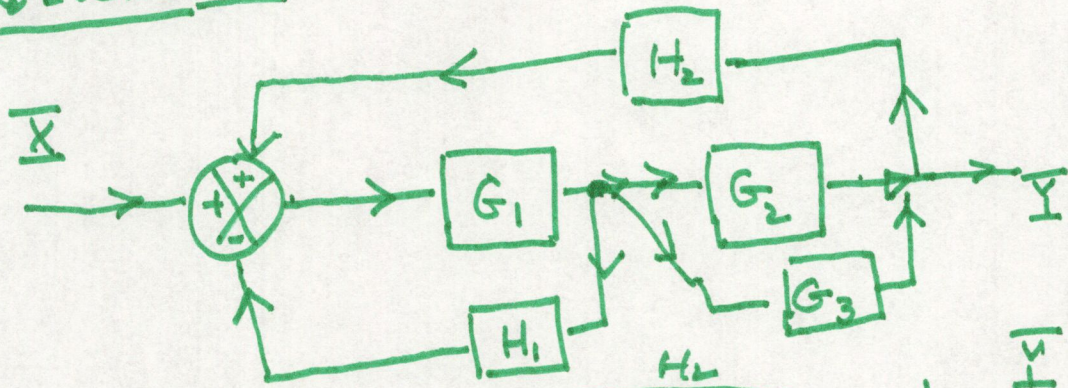
$L_1 L_2 \times$
 $L_1 L_3 \times$
 $L_2 L_3 \times$

$L_1 L_2 L_3 \times$

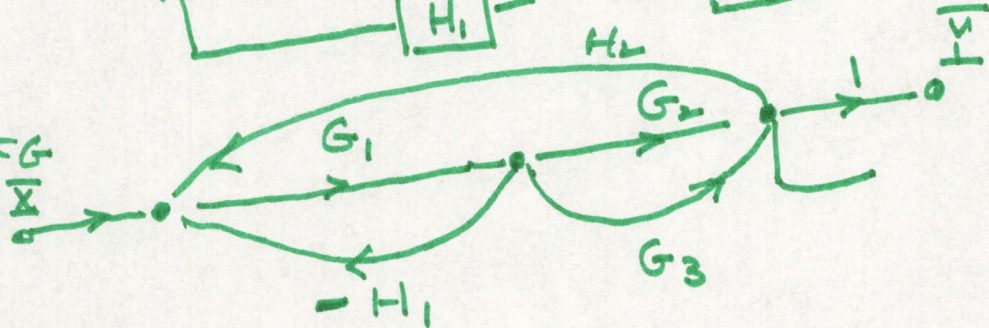
$\Delta_1 = 1$

$\frac{Y}{X} = \frac{P_1 \Delta_1 + P_2 \Delta_2}{\Delta}$

Problem soln



A. SFG



B. Find $\frac{Y}{X}$

Forward path gain

$$P_1 = G_1 G_2$$

$$P_2 = G_1 G_3$$

1 Path/loop minus

Loops

$$L_1 = H_2 G_1 G_2$$

$$L_2 = -H_1 G_1$$

$$\Delta = 1 - (L_1 + L_2) + (L_1 L_2)$$

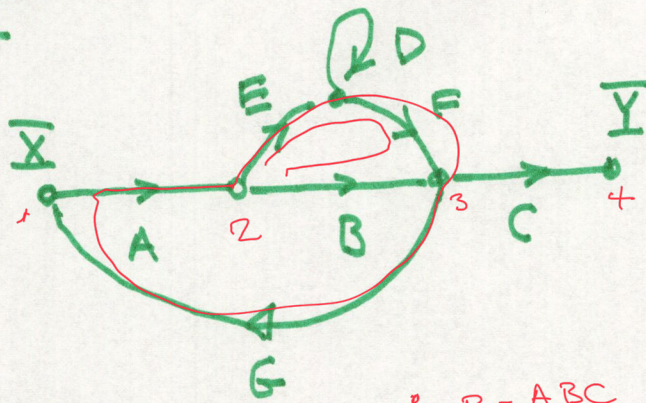
touch yes

$$\Delta_1 = 1$$

$$\Delta_2 = 1$$

$$\frac{Y}{X} = \frac{P_1 \Delta_1 + P_2 \Delta_2}{\Delta}$$

Problem



Find $\frac{Y}{X}$

Loops $\Rightarrow L_1 = D$

$\bullet \bullet \bullet \bullet$



$\Rightarrow L_2 = ABG$



$\bullet L_3 = AEEFG$

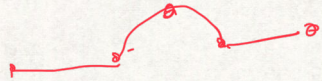
$\Delta_2 = 1$

$\left[\begin{array}{ccc} L_1 & L_2 & L_3 \end{array} \right]$

$\bullet P_1 = ABC$



$P_2 = AEFB$



$L_1 L_2 \checkmark$

$L_1 L_3 \times$

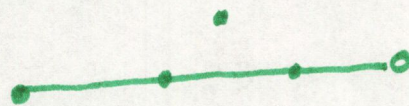
$L_2 L_3 \times$

$$\Delta = 1 - (L_1 + L_2 + L_3) + L_1 L_2$$

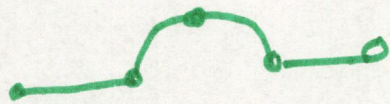
$$\Delta_1 = 1 - (L_1) \quad \frac{Y}{X} = \frac{P_1 \Delta_1 + P_2 \Delta_2}{\Delta}$$

Problem soln

Forward Path

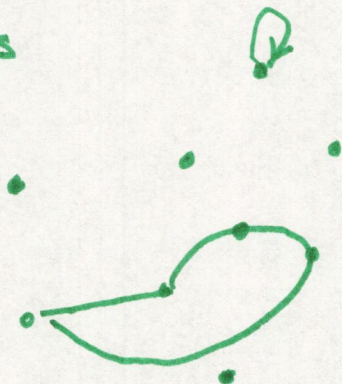


$$P_1 = ABC$$

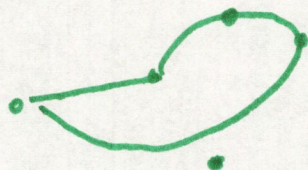


$$P_2 = AEFC$$

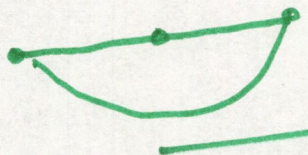
Loops



$$L_1 = D$$



$$L_2 = AEFG$$



$$L_3 = ABG$$

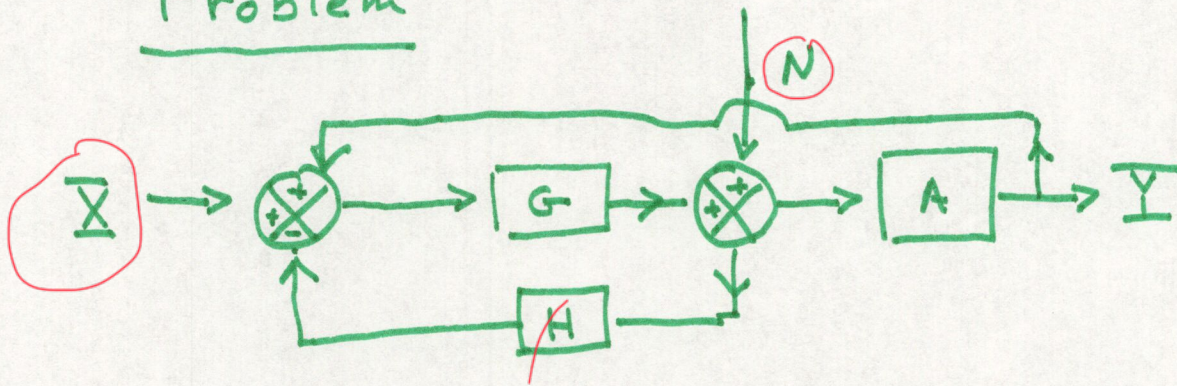
$$\Delta = 1 - (L_1 + L_2 + L_3) + \begin{bmatrix} ? \\ \uparrow \end{bmatrix} \begin{matrix} L_1 L_2 \\ L_1 L_3 \end{matrix} \begin{matrix} \times \\ \checkmark \end{matrix} \left. \begin{matrix} \text{two} \\ \text{at a time} \end{matrix} \right\} - \begin{bmatrix} ? \\ \uparrow \end{bmatrix} \begin{matrix} L_2 L_3 \\ L_1 L_2 L_3 \end{matrix} \begin{matrix} \times \\ \times \end{matrix} \left. \begin{matrix} \text{three} \\ \text{at a time} \end{matrix} \right\}$$

$$\Delta_1 = 1 - L_1$$

$$\Delta_2 = 1$$

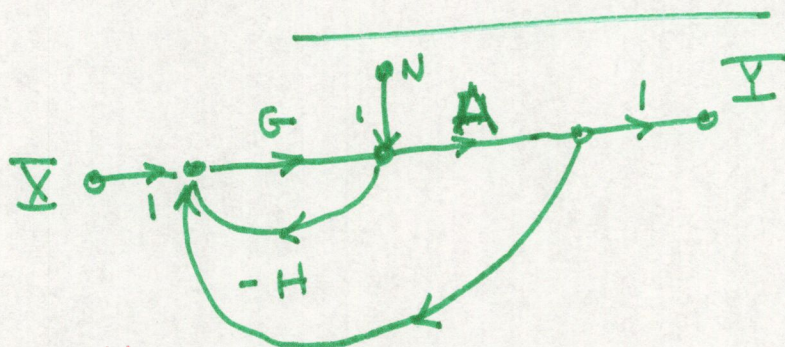
$$\bar{Y} = \frac{P_1 \Delta_1 + P_2 \Delta_2}{\Delta}$$

Problem

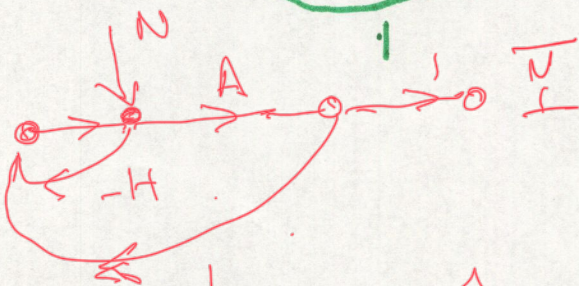


Find $\bar{Y}$

$$\bar{Y} = \left(\frac{\bar{Y}}{\bar{X}} \right) \bigg|_{N=0} \bar{X} + \left[\frac{\bar{Y}}{\bar{N}} \right] \bigg|_{\bar{X}=0} \bar{N}$$

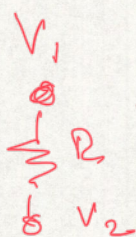


$$\left. \frac{\bar{Y}}{\bar{X}} \right|_{N=0} = \frac{P_1}{1 + (L_1 + L_2)}$$

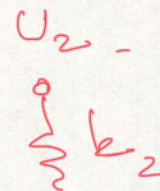


$$P_1 = A$$

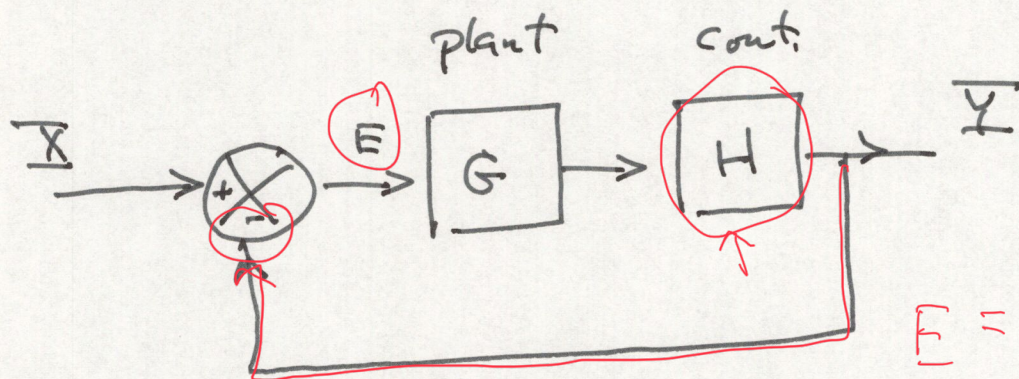
$$L_1 = -HG$$



=



Steady state error



$$X \rightarrow [G] \rightarrow Y$$

obj

$$\underline{X - Y = 0}$$

$$E = X - Y$$

$$\left. \begin{aligned} \frac{Y}{X} &= \frac{GH}{1+GH} \\ E &= X - Y \end{aligned} \right\}$$

$$\frac{E}{X} = 1 - \frac{Y}{X} = \frac{1}{1+G}$$

$$E = X - Y$$

$$\frac{E}{X} = 1 - \frac{Y}{X}$$

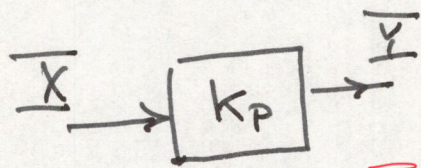
$$\Rightarrow E(s) = \frac{1}{1+GH} X$$

$$\frac{E}{X} = 1 - \frac{GH}{1+GH}$$

$$\frac{E}{X} = \frac{1+GH-GH}{1+GH}$$

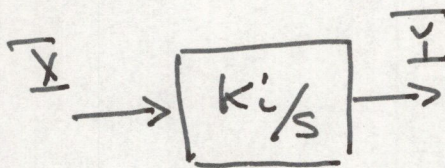
$$\left[\frac{E}{X} = \frac{1}{1+GH} \right]$$

Control Action



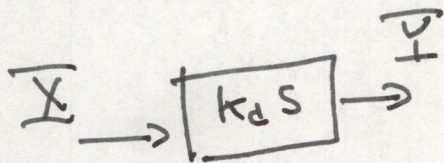
$$\bar{Y} = k_p \bar{X}$$

prop-cont



$$\bar{Y} = \frac{k_i}{s} \bar{X}$$

integral cont



$$\bar{Y} = k_d s \bar{X}$$

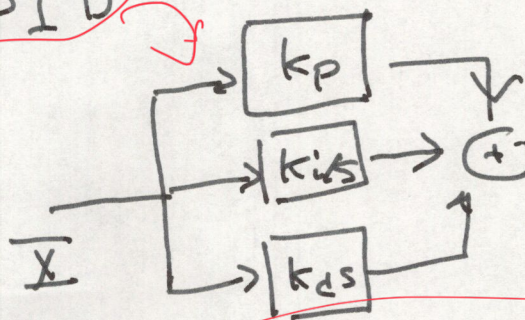
deriv-cont

$$y = k_i \int x dt$$

$$y = k_d \frac{dx}{dt}$$

PID

PI



$$\bar{Y} = \left[k_p + \frac{k_i}{s} + k_d s \right] \bar{X}$$

adjust k_p, k_i, k_d

Final value theorem

$$\lim_{s \rightarrow 0} s E(s) = \lim_{t \rightarrow \infty} e(t)$$

where $E(s) \xleftrightarrow{\mathcal{L}} e(t)$