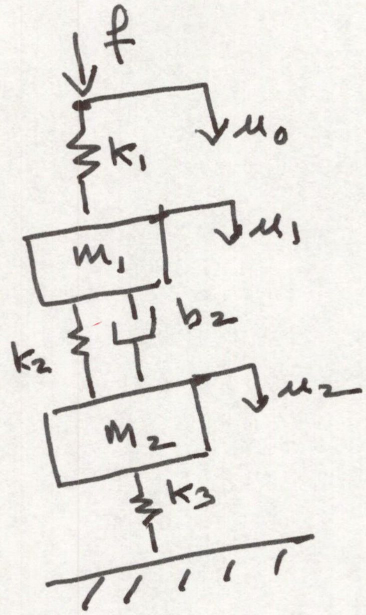
Problem

Consider



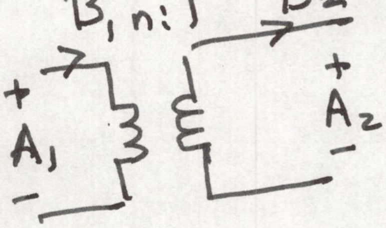
(a) Find Electro model

(b) Eqn of motion in Laplace domain

(c) Eqn of motion in time-domain

(d) U_2/U_1

The transform



$A_1 B_1 = A_2 B_2$

$B_1 n = B_2$

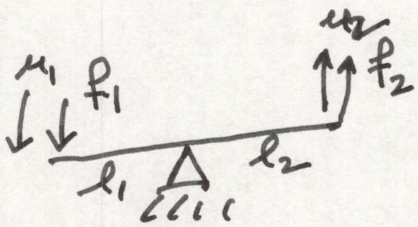
$A_1 / n = A_2$

Electrical

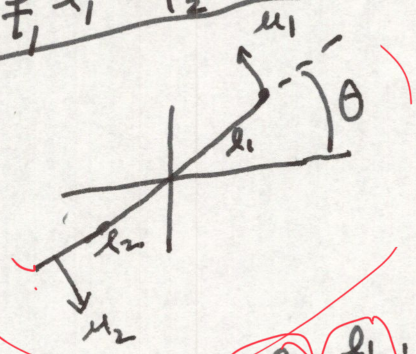
$I_1 n = I_2$

$V_1 / n = V_2$

Mechanical



$\boxed{f_1 l_1 = f_2 l_2}$



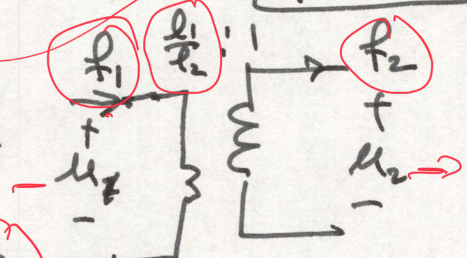
$$\left.\begin{aligned} u_1 &= \dot{\theta} l_1 \\ u_2 &= \dot{\theta} l_2 \end{aligned}\right\}$$

$$\boxed{\frac{u_1}{l_1} = \frac{u_2}{l_2}}$$

$l_2 \dot{\theta} = u_2$

$l_1 \dot{\theta} = u_1$

$\dot{\theta} = \frac{u_2}{l_2} = \frac{u_1}{l_2}$



$f_1 \frac{l_1}{l_2} = f_2$

$l_2 \dot{\theta} = u_2$

$u_1 \frac{l_2}{l_1} = u_2$

Acoustical Elements

Newton's Law

force$/m^3$ =

$$\rho_0 \frac{\partial u}{\partial t} + \frac{\partial P}{\partial x} = 0 \quad (1)$$

Cons of mass

$$\frac{1}{c^2}\frac{\partial P}{\partial t} + \rho_0 \frac{\partial u}{\partial x} = 0 \quad (2)$$

where ρ_0 is the density of air (kg/m^3) 1.14

c is the speed of sound $c = 345\ m/s$

u particle velocity (m/s)

P pressure (Nt/m^2)

$$\frac{\partial P}{\partial x} = \frac{Nt}{m^3}$$

Wave eqn

$$\frac{\partial (1)}{\partial x} - \frac{\partial}{\partial t}(2) = 0$$

$$\boxed{\frac{\partial^2 P}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 P}{\partial t^2} = 0}$$

Soln

$$p(x,t) = \left[P^+ e^{-i\frac{\omega}{c}x} + P^- e^{i\frac{\omega}{c}x}\right] e^{i\omega t}$$

using (1)

$$u(x,t) = \frac{1}{\rho_0 c}\left[P^+ e^{-i\frac{\omega}{c}x} - P^- e^{i\frac{\omega}{c}x}\right] e^{i\omega t}$$

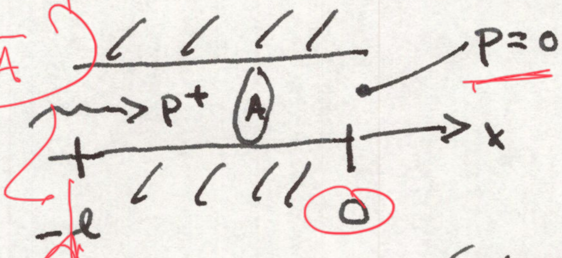
$P^- = -P^+$

$$p(0,t) = \left[P^+ + P^-\right] e^{i\omega t} = 0$$

$$u(0,t) = \left[\frac{1}{\rho_0 c}\right]\left[P^+ - P^-\right] e^{i\omega t}$$

Acoustic mass

$Z_A = \frac{p}{uA}$



free

$$p(0,t) = (p^+ + p^-)\big| = 0$$

$$\therefore\ p^- = p^+(-1)$$

then

$$p(x,t) = p^+\left[e^{-i\frac{\omega}{c}x} - e^{i\frac{\omega}{c}x}\right]e^{i\omega t}$$

$$u(x,t) = \frac{p^+}{\rho_0 c}\left[e^{-i\frac{\omega}{c}x} + e^{i\frac{\omega}{c}x}\right]e^{i\omega t}$$

$$p(x,t) = p^+(-2j)\sin\left(\frac{\omega}{c}x\right)e^{i\omega t}$$

$$u(x,t) = \frac{p^+}{\rho_0 c}(2)\cos\left(\frac{\omega}{c}x\right)e^{i\omega t}$$

$e^{i\theta} + e^{-i\theta} =?\ 2\cos\theta$

$\frac{e^{i\theta} + e^{-i\theta}}{2} = \cos\theta$

$\frac{e^{i\theta} - e^{-i\theta}}{2j} = \sin\theta$

the Acoustic impedance

$$\frac{P}{AU} = \frac{P}{Q}\Big|_{x=-l} = \frac{\rho_0 c}{A} j \frac{\sin\left(\frac{\omega}{c}l\right)}{\cos\left(\frac{\omega}{c}l\right)}$$

$\frac{P}{UA}\Big|_{x=-l}$

$0 \leftarrow x$, $\sin(x) \sim x$

Q is the volume velocity

$\frac{\omega}{c}l \to 0$ $\quad = \frac{j\rho_0 c}{A}\left[\frac{\frac{\omega}{c}l}{1}\right]$

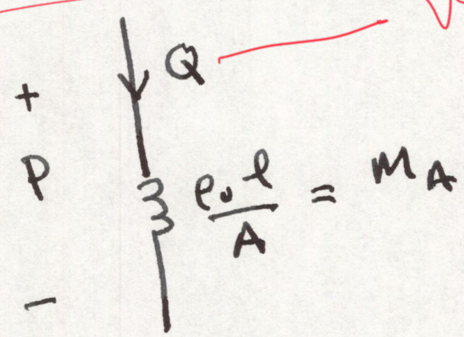
long wavelength approx

$$\frac{\omega}{c} l = 2\pi \frac{l}{\lambda}$$

$$\frac{W}{C} l = 2\pi \frac{l}{\lambda}$$

$$\lim_{\frac{\omega l}{c} \to 0} j \frac{\rho_0 c}{A} \frac{\sin\left(\frac{\omega}{c} l\right)}{\cos\left(\frac{\omega}{c} l\right)} = j \frac{\rho_0 c}{A} \frac{\omega l}{c}$$

$$\boxed{\frac{P}{Q} = j\omega \frac{\rho_0 l}{A}}$$



volume velocity

$u A$

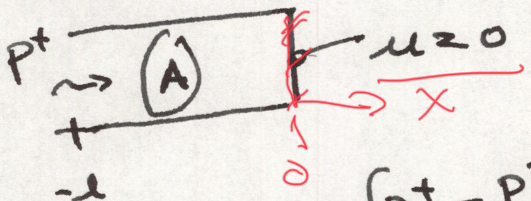
$$\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!}$$

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!}$$

$$\lim_{x \to 0} \cos(x) = 1$$

$$\lim_{x \to 0} \sin(x) = x$$

Acoustic Compliance



$$u(0,t) = \left[p^{+} - p^{-}\right] \frac{1}{\rho_0 c} e^{i\omega t} = 0$$

$$\boxed{p^{-} = p^{+}}$$

$$p(x,t) = p^{+}\left[e^{-j\frac{\omega}{c}x} + e^{j\frac{\omega}{c}x}\right] e^{i\omega t}$$

$$u(x,t) = \frac{p^{+}}{\rho_0 c}\left[e^{-j\frac{\omega}{c}x} - e^{j\frac{\omega}{c}x}\right] e^{i\omega t}$$

$$p(x,t) = p^{+}\, 2\cos\left[\frac{\omega}{c}x\right] e^{i\omega t}$$

$$u(x,t) = \frac{p^{+}}{\rho_0 c}(-2j)\sin\left[\frac{\omega}{c}x\right] e^{i\omega t}$$

$$Z_A = \left.\frac{p}{uA}\right|_{x=-l} = \frac{\rho_0 c}{jA} \frac{\cos\left[\frac{\omega}{c}l\right]}{\sin\left[\frac{\omega}{c}l\right]} \qquad \frac{\omega}{c}l \to 0$$

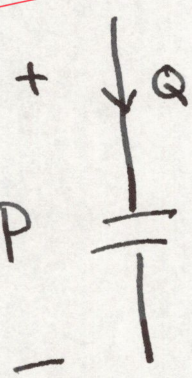
$$= \frac{\rho_0 c}{jA}\left[\frac{1}{\frac{\omega}{c}l}\right]$$

long wave length limit

$$\frac{\omega}{c} l \to 0$$

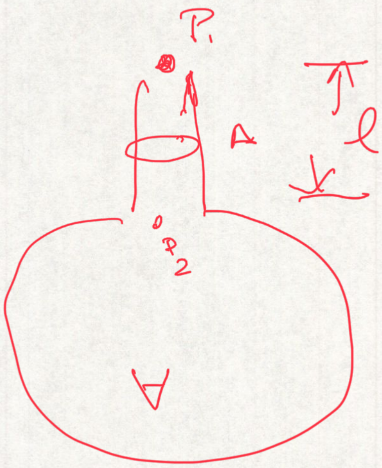
$$Z_A = \frac{P}{Q}\bigg|_{x=-l} \simeq \frac{\rho_0 c}{jA} \frac{1}{\frac{\omega l}{c}}$$

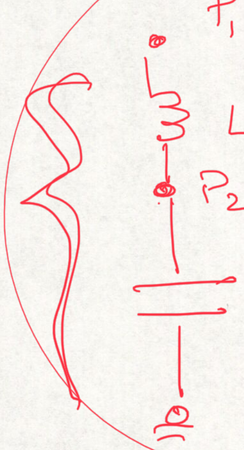
$$Z_A = \frac{\rho_0 c^2}{A l} \frac{1}{j\omega}$$



$$C_A = \frac{A l}{\rho_0 c^2}$$

∀olume



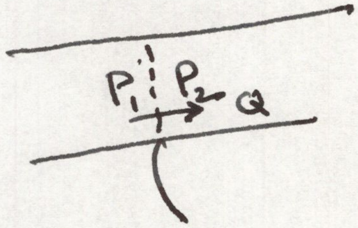


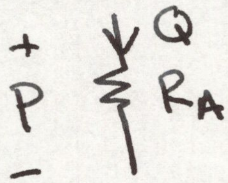
$$L_A = \frac{\rho_0 l}{A}$$

$$C_A = \frac{\forall}{\rho_0 c^2}$$

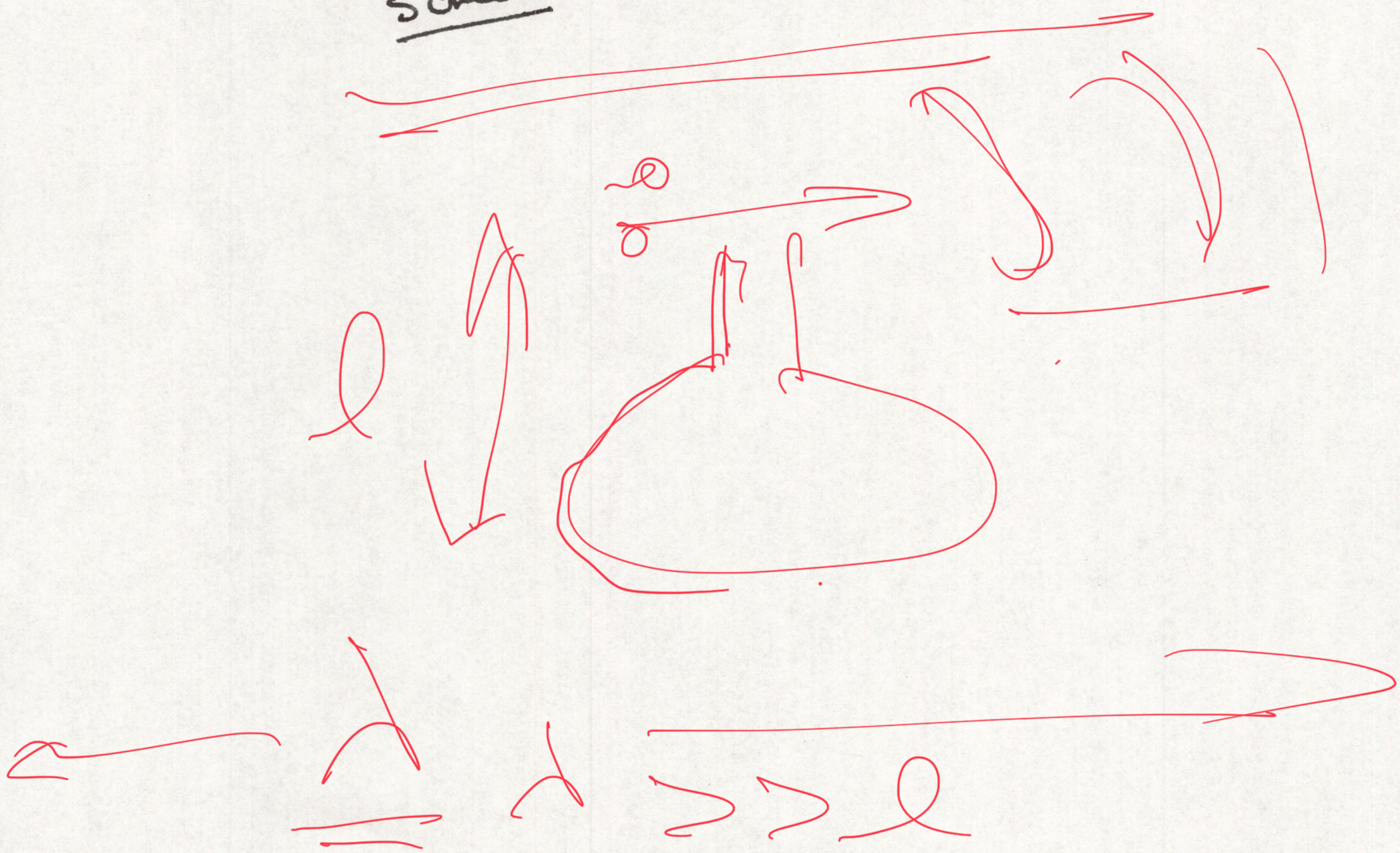
≃ tuned exhaust

Acoustic resistance



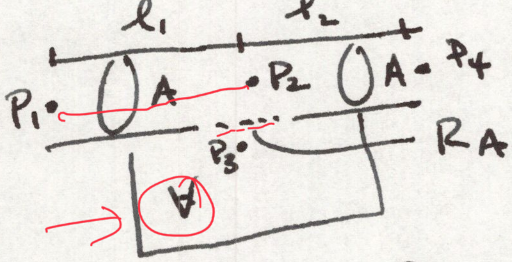


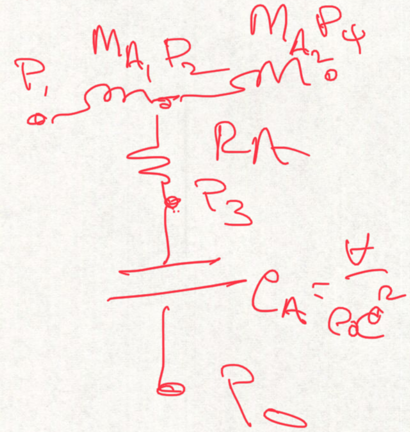
Screen

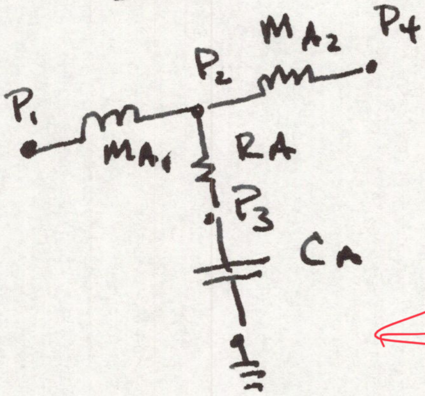


Example

muffler







$$C_A = \frac{\forall}{\rho_0 c^2}$$

$$M_{A_1} = \frac{\rho_0 l_1}{A}$$

$$M_{A_2} = \frac{\rho_0 l_2}{A}$$