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Inhomogeneous Solution

$$\dot{\underline{x}} = A\underline{x} + B\underline{u}$$

$$\underline{x}(t) = \Phi \underline{x}(0) + \int_{-\infty}^t \Phi(t-\tau) B \underline{u}(\tau) d\tau$$

proof:

$$\dot{\underline{x}} = A\underline{x} + B\underline{u}$$

taking the Laplace transform

$$[s\underline{X} - \underline{x}(0)] = A\underline{X} + B\underline{U}$$

$$[sI\underline{X} - \underline{x}(0)] = A\underline{X} + B\underline{U}$$

$$[sI - A]\underline{X} = \underline{x}(0) + B\underline{U}$$

$$\underline{X} = [sI - A]^{-1} \underline{x}(0) + [sI - A]^{-1} B \underline{U}$$

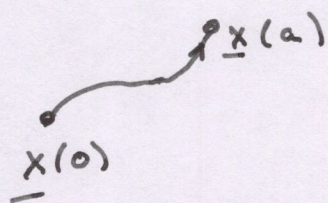
$$[sI - A]^{-1} \leftrightarrow \Phi$$

L-trans pair

Invert

$$\underline{x} = \Phi \underline{x}(0) + \Phi * B \underline{u}$$

Controllable



Controllable if
if one can start
at any $x(0)$
a travel to any location
 $x(a)$ under the
action of a
input u

How can one
determine if a system
is controllable?

Given $\dot{x} = Ax + Bu$
where A has unique eigenvalues

$$\text{let } x = Ez$$

$$E\dot{z} = AEz + Bu$$

$$\dot{z} = E^{-1}AEz + E^{-1}Bu$$

$$\dot{z} = \Lambda z + \underbrace{(E^{-1}B)}_{\text{matrix}} u$$

If any row of $E^{-1}B$ is all zero
the system is not controllable

example

$$\dot{\underline{z}} = \underline{A} \underline{z} + \underline{E}^{-1} \underline{B} \underline{u}$$

$$\textcircled{1} \begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 & 0 & 0 \\ 6 & 1 & 7 \end{bmatrix}}_{\text{all zero not controllable}} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

note $\begin{bmatrix} 0 & 0 & 0 \\ 6 & 1 & 7 \end{bmatrix} \leftarrow$ all zero not controllable

$$\begin{aligned} \dot{z}_1 &= \lambda_1 z_1 \\ \dot{z}_2 &= \lambda_2 z_2 + 6u_1 + 1u_2 + 7u_3 \end{aligned} \quad \leftarrow \text{not function } \underline{u}$$

$$\textcircled{2} \dot{\underline{z}} = \underline{A} \underline{z} + \begin{bmatrix} a & 0 & b \\ 0 & c & d \\ q & 1 & r \end{bmatrix} \underline{u}_1$$

not controllable if
 $a=b=0$
~~or~~
 or
 $c=d=0$

Observability

$$\dot{\underline{x}} = A \underline{x}$$

$$\underline{y} = C \underline{x}$$

$$\text{let } E \underline{z} = \underline{x}$$

$$\dot{\underline{z}} = A \underline{z}$$

$$\underline{y} = (CE) \underline{z}$$

if any column of CE is all
zero the system is
not obs.

Observable

if one can ~~be~~
determine $\underline{x}(0)$
from observation
of $\underline{y}$ over a
finite interval
of time

example

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$$

$$\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} e^{\lambda_1 t} z_1(0) \\ e^{\lambda_2 t} z_2(0) \end{bmatrix}$$

$$y = \begin{bmatrix} 0 & 1 \\ 0 & 2 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} \Rightarrow \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} z_2 \\ 2z_2 \\ 3z_2 \end{bmatrix}$$

$\uparrow$
 not
obs

$$y_1 = z_2$$

$$y_2 = 2z_2$$

$$y_3 = 3z_2$$

note z_1 is not obs
by examining
 y_1, y_2, y_3

Example

$$\dot{\underline{z}} = \Lambda \underline{z}$$

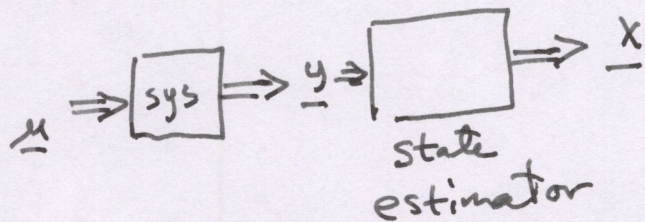
$$\underline{y} = (CE) \underline{z}$$

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} \Rightarrow \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} z_1(0)e^{\lambda_1 t} \\ z_2(0)e^{\lambda_2 t} \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 \\ 0 & 2 \\ 0 & 3 \end{bmatrix}}_{CE} \begin{bmatrix} z_1(t) \\ z_2(t) \end{bmatrix} \Rightarrow \begin{aligned} y_1 &= z_2(t) \\ y_2 &= 2z_2 \\ y_3 &= 3z_2 \end{aligned}$$

z_1 is not obs at output

therefore if obs



using a state est one
can find $\underline{x}$ given $\underline{y}$

if controllable given $\underline{x}$ one can
find $\underline{u}$ using control action

