Root Locus

1° RL - RL $\left\{ \begin{array}{l} \text{id} \quad \text{breakaway} \\ \qquad\quad \text{breakin} \quad \text{region} \\ \text{direction of } \cancel{\text{migratn}} \text{ migratn} \end{array} \right.$

2° asym/intercept $\left\{ \begin{array}{l} \text{id} \quad j\omega\text{-axis} \\ \qquad\quad \text{crossing} \end{array} \right.$

$$\sigma = \frac{\sum \text{poles} - \sum \text{zeros}}{\#\text{poles} - \#\text{zeros}}$$

$$\measuredangle = \pm \frac{(2n+1)\,\pi}{\#\text{poles} - \#\text{zeros}}$$

3° $j\omega$-axis crossing (via Routh Table)

using $1 + GH = 0$

$$1 + k\frac{N(s)}{D(s)} = 0$$

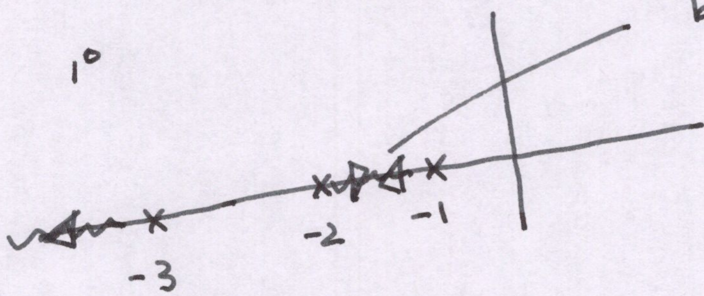
$$\boxed{D(s) + kN(s) = 0}$$

example

$$GH = \frac{K}{(s+1)(s+2)(s+3)}$$

breakaway pt

1°

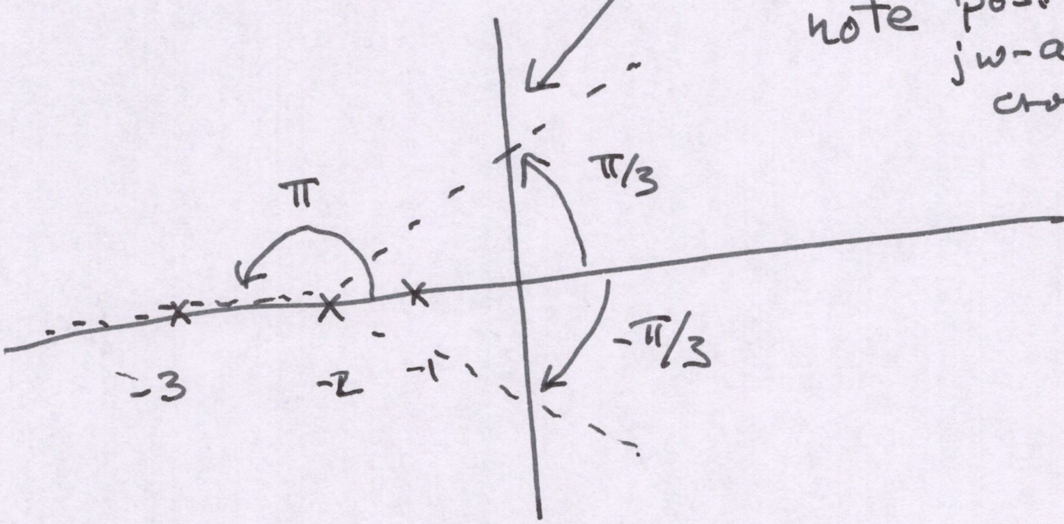


2°

$$\sigma = \frac{-3-2-1-[0]}{3-0} = -\frac{6}{3} = -2$$

$$\measuredangle = \frac{\pm(2n+1)\pi}{3-0} = \begin{cases} \pm\pi/3 & n=0 \\ \pm\pi & n=1 \end{cases}$$

note possible jw-axis crossing



3° jw-axis crossing

$1+GH=0 \Rightarrow 1+\frac{k}{(s+1)(s+2)(s+3)}=0$

$$\boxed{s^3+6s^2+11s+6+k=0}$$

s^3	1	11
s^2	6	$6+k$
s^1	$\frac{(6)(11)-(1)(6+k)}{6}$	
s^0	$6+k$	

$\rightarrow \frac{66-6-k}{6}=\frac{60-k}{6}$

stable

$\frac{60-k}{6}>0$ $k<60$ $\Big\}$ $-6<k<60$

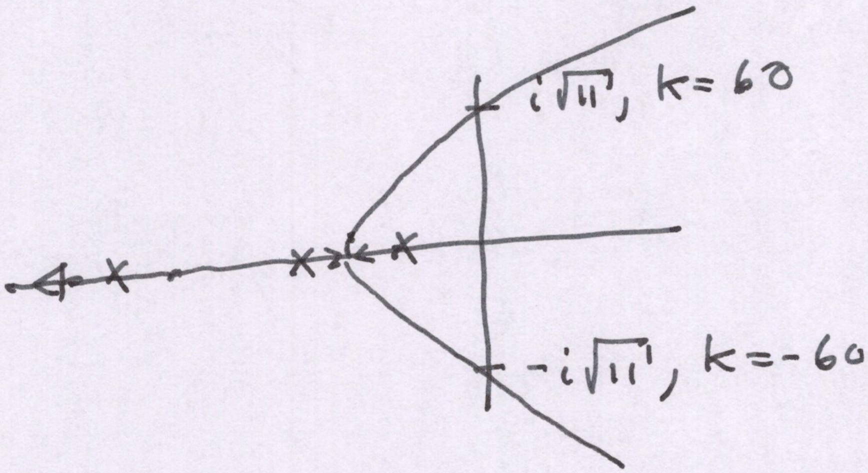
$6+k>0 \Rightarrow k>-6$

jw-axis crossing

$k=60$ $\boxed{k=60}$

aux poly: $6s^2+6+k=0$;

$$\boxed{s=\pm i\sqrt{11}}$$



4. Breakaway Pt / Breakin Pt

$$GH + 1 = 0$$

$$k\frac{N}{D} + 1 = 0$$

$$k = -\frac{D}{N}$$

$$\frac{dk}{ds} = -\frac{d}{ds}\left[\frac{D}{N}\right] = 0$$

$$= -\left[\frac{D'}{N} - \frac{N'D}{N^2}\right] = 0$$

$$\Rightarrow \boxed{D'N - N'D = 0}$$

Breakin / Breakaway are the roots of $\frac{dk}{ds} = 0$

example

$$s^3+6s^2+11s+6+k=0$$

$$\frac{dk}{ds}=-\left[3s^2+12s+11\right]=0$$

−2.56

$$s=\begin{cases} -\dfrac{\sqrt{3}+6}{3} & \Leftarrow \cancel{\text{pick}} \\[2ex] \dfrac{\sqrt{3}-6}{3}=-2+\dfrac{\sqrt{3}}{3} & \Leftarrow \text{pick} \end{cases}$$

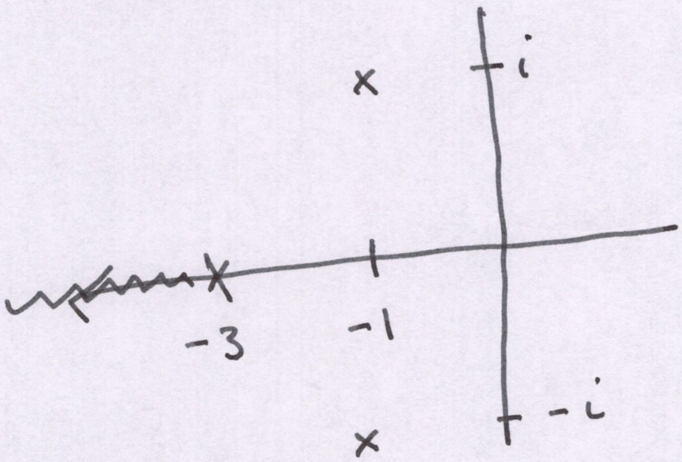
−1.4

$$k=-\left[s^3+6s^2+11s+6\right]\Big|_{s=-1.4}$$

$$\boxed{k=\frac{48}{125}}$$

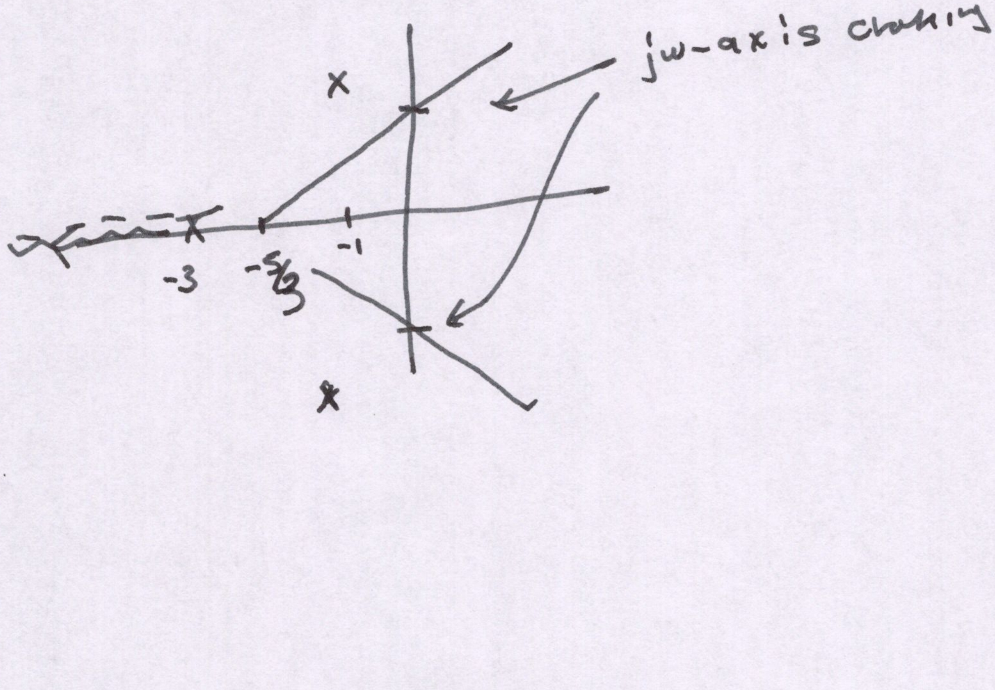
5. Angle of departure/arrival

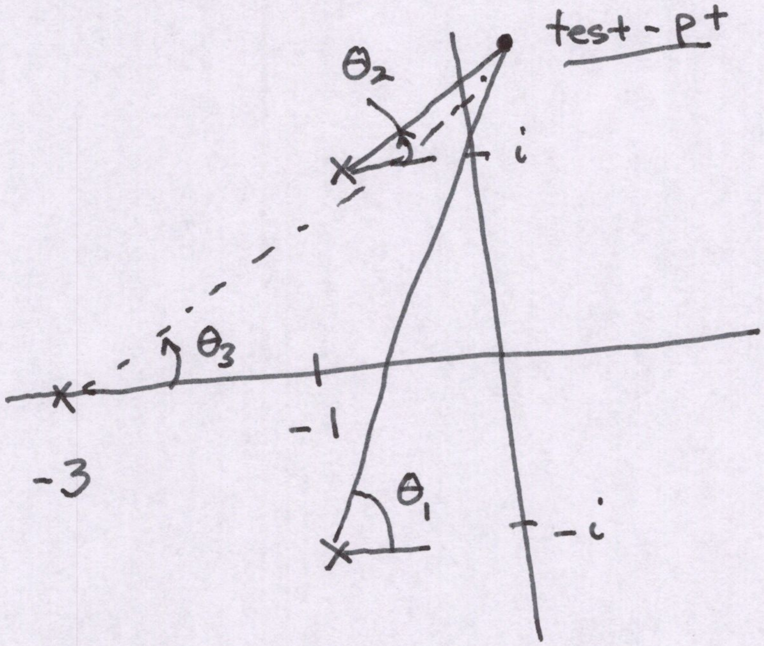
(cmplx conjugate pairs)



$$\sigma = \frac{-3 - 1 + i - 1 - i \; - [0]}{3 - 0} = -\frac{5}{3}$$

$$\measuredangle = \pm\frac{(2n+1)\pi}{3} = \begin{cases} \pm\pi/3 \\ \pm\pi \end{cases}$$





$$\measuredangle = -\sum \measuredangle \text{poles} + \sum \measuredangle \text{zeros} + \measuredangle K = \pm(2n+1)\pi$$

$$-\theta_2 - \theta_1 - \theta_3 + 0 = \pm(2n+1)\pi$$

more testpt to <u>pole</u>.

$$-\theta_2 - \frac{\pi}{2} - \tan^{-1}\left(\frac{1}{2}\right) = \pi$$

$$\theta_2 = -\frac{\pi}{2} - \tan^{-1}\left(\frac{1}{2}\right) + \pi$$

angle of departure