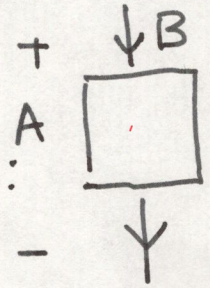
Modeling

A. Ideal Element



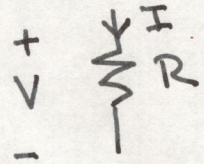
A across variable

B through variable

such that

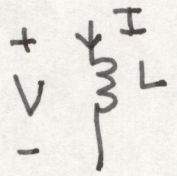
$$\frac{AB^*}{2} = \text{Power}$$

Electrical Elements



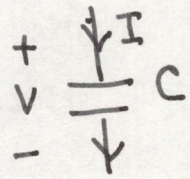
Laplace-domain: $V = IR$

time-domain: $v = iR$



$V = LsI$

$v = L\frac{di}{dt}$

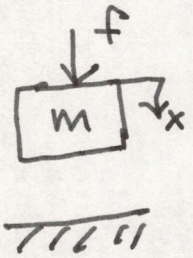


$V = \frac{I}{Cs}$

$v = \frac{1}{C}\int i\,dt$

Rectilinear Mechanical

A. The mass



$f = m\ddot{x}$ ⇓ $f = m\dot{u}$

x displacement from rest

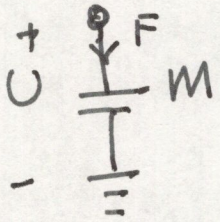
u velocity

$f u$ = Power

(i) which is the across variable F or U?

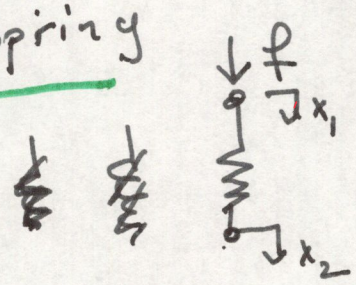
(ans) U due to consideration of the interial reference frame

$f = m\dot{u} \xrightarrow{\mathcal{L}} F = MsU \Rightarrow \boxed{U = \frac{1}{Ms} F}$



hence all masses are tided to ground!!

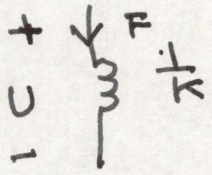
B. the Spring



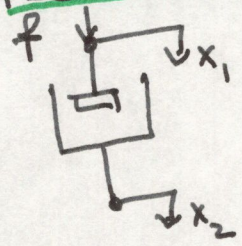
$$f = k(x_1 - x_2)$$

$$f = k\int (v_1 - v_2)\,dt$$

$$F = k\frac{1}{s}(v_1 - v_2) \Rightarrow [v_1 - v_2] = \boxed{\frac{s}{k}F = V}$$



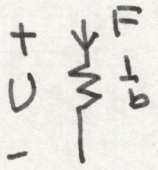
C. the damper



$$f = b(\dot{x}_1 - \dot{x}_2)$$

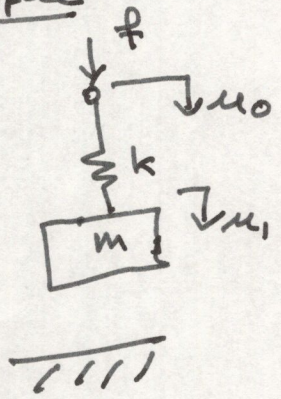
$$f = b(v_1 - v_2) \Rightarrow \boxed{F = bV}$$

$$\boxed{V = \frac{F}{b}}$$



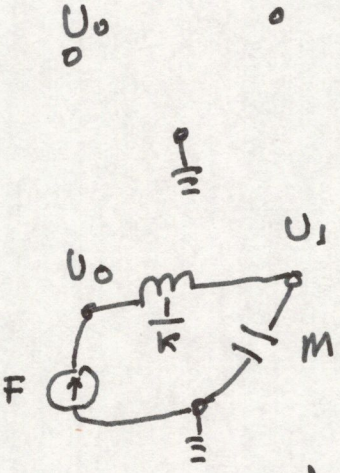
Example

given



1. Find Electro-mech circuit with u is the across variable

Step 1:



Step 2:

2. Find eqn of motion in Laplace domain

KCL

node U_0: $F = \frac{U_0 - U_1}{\frac{s}{K_s}}$

node U_1 $\frac{U_0 - U_1}{\frac{s}{K}} = \frac{U_1}{\frac{1}{Ms}}$

3. Find the governing ode's of the system

$$F = \frac{u_0 - U_1}{s/k} \Rightarrow sF = k[U_0 - U_1] \Rightarrow \boxed{\dot{f} = k[u_0 - u_1]} \quad (1)$$

$$\frac{U_0 - U_1}{\frac{s}{k}} = \frac{U_1}{\frac{1}{Ms}} \Rightarrow k[U_0 - U_1] = Ms^2 U_1 \Rightarrow \boxed{k[u_0 - u_1] = M\ddot{u}_1} \quad (2)$$

4. Find the transfer function $\frac{U_1(s)}{U_0(s)}$

ie find $U_1(s)$ given $U_0(s)$

$$k[U_0 - U_1] = Ms^2 U_1 \Leftarrow (2)$$

$$\Downarrow$$

$$k[U_0] = [Ms^2 + k] U_1$$

$$\boxed{\frac{U_1}{U_0} = \frac{k}{ms^2 + k}}$$

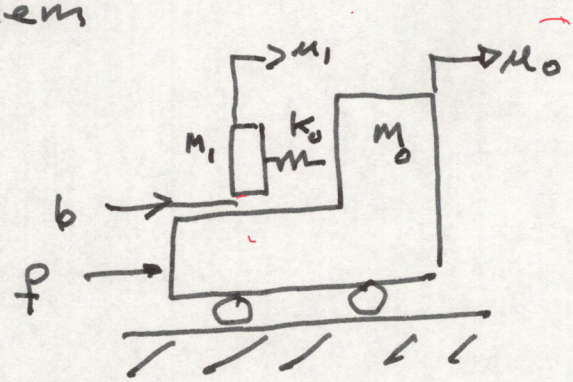
$\propto$ +k

5. find asymp. behavior at

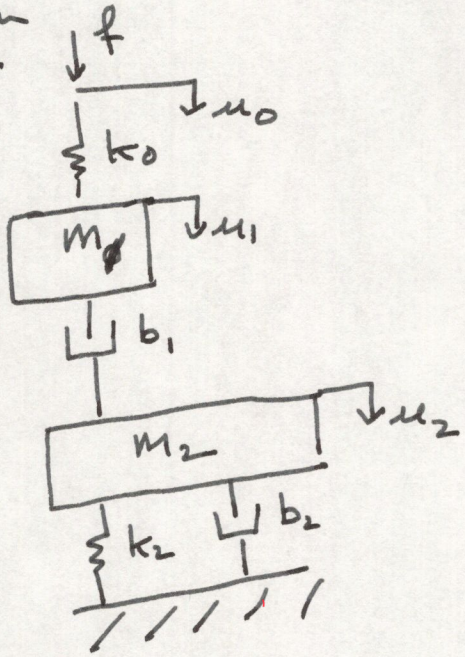
(a) $\omega \to 0$

(b) $\omega \to \infty$

Problem



Problem



Find EM circuit
u across variable

Problem

Given