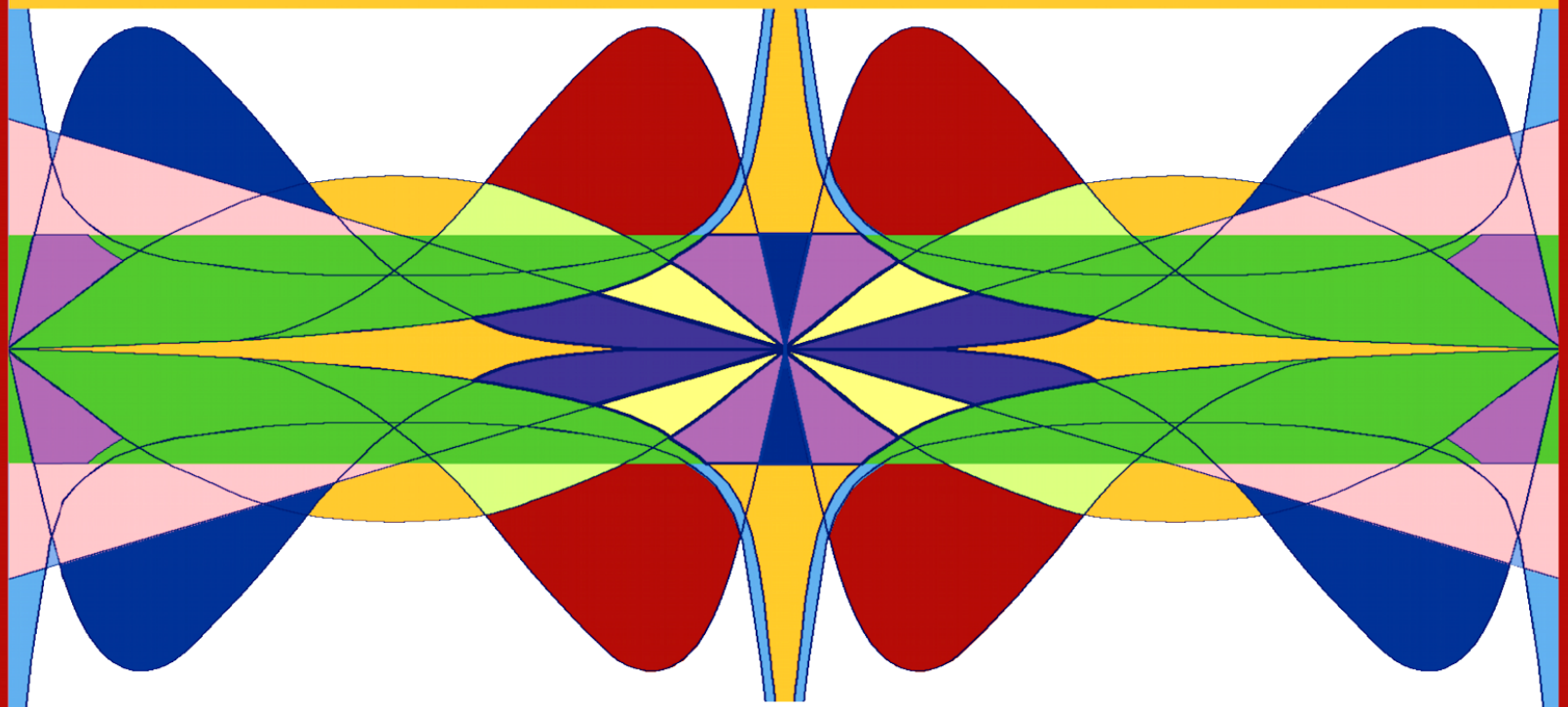
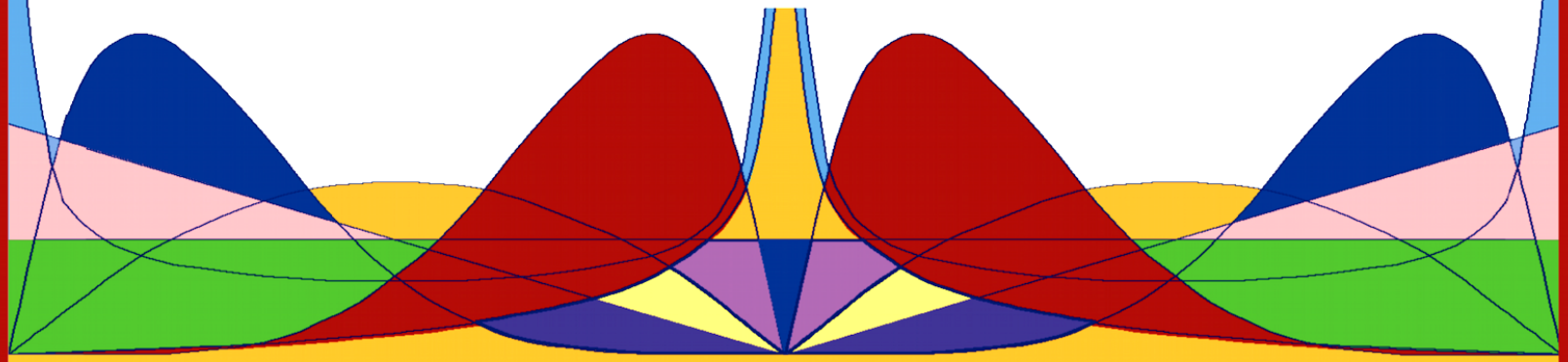
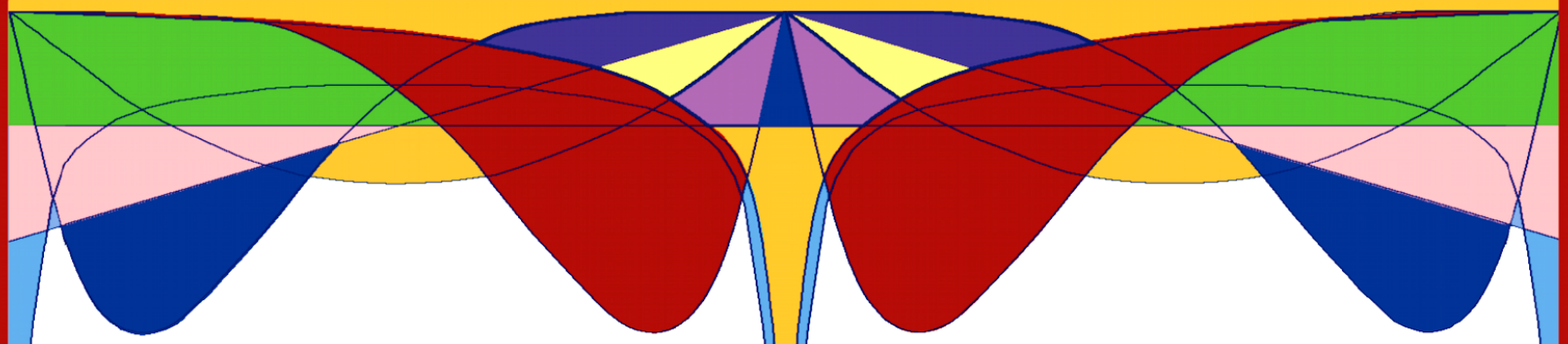
SECOND EDITION



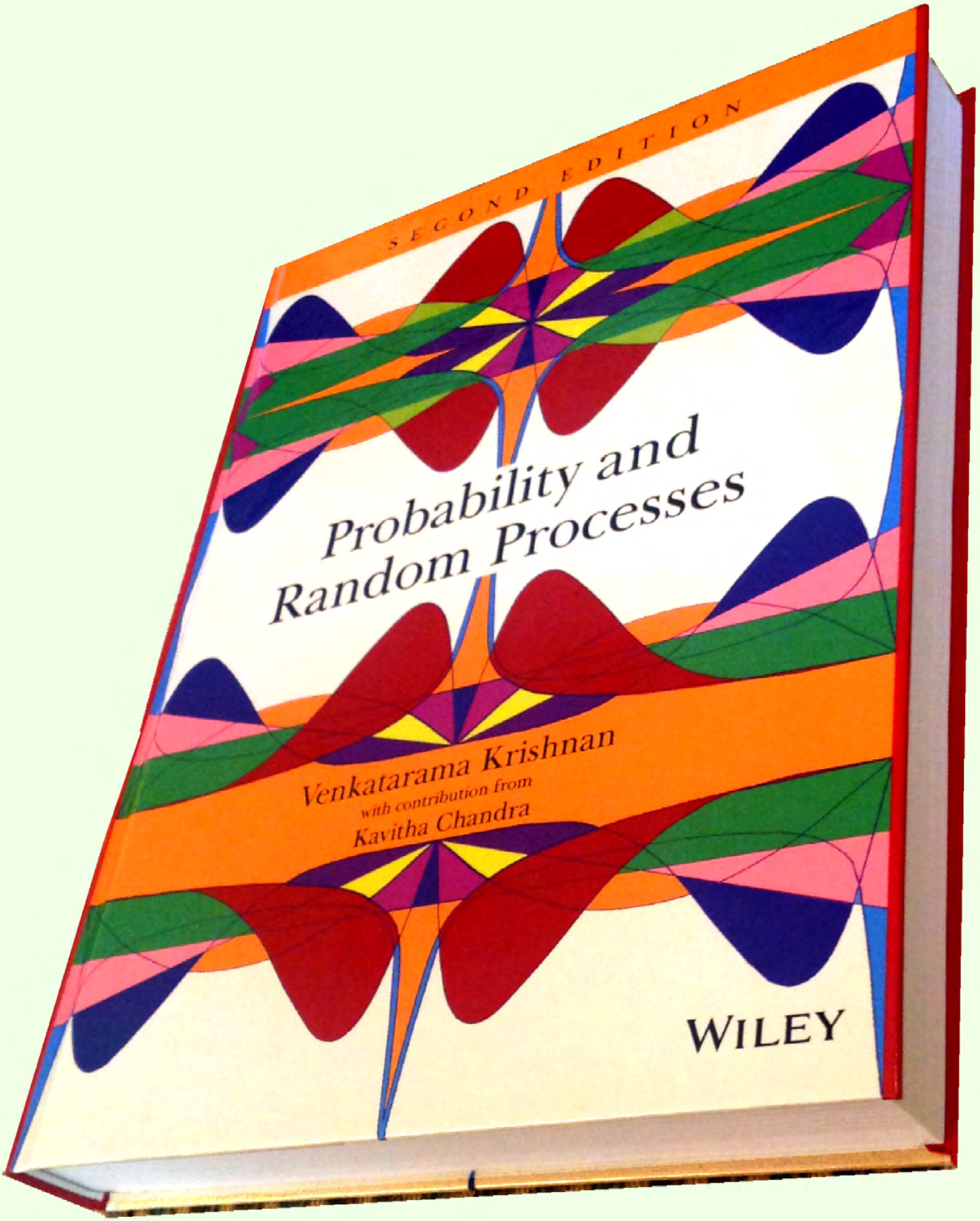
Probability and Random Processes



Venkatarama Krishnan
with contribution from
Kavitha Chandra



WILEY


SECOND EDITION
Probability and Random Processes
Venkatarama Krishnan
with contribution from
Kavitha Chandra
WILEY

PROBABILITY AND RANDOM PROCESSES

Published in July, 2015

PROBABILITY AND RANDOM PROCESSES

SECOND EDITION

VENKATARAMA KRISHNAN
Professor Emeritus of Electrical and Computer Engineering
Center for Advanced Computation and Telecommunications
University of Massachusetts Lowell

With Contribution from

KAVITHA CHANDRA
Professor of Electrical and Computer Engineering
Center for Advanced Computation and Telecommunications
University of Massachusetts Lowell

WILEY

Published by John Wiley & Sons, Inc., Hoboken, New Jersey
Published simultaneously in Canada

Library of Congress Cataloging-in-Publication Data:

Krishnan, Venkatarama, 1929-
Probability and random processes/ Venkatarama Krishnan - Second Edition
pages 522
Includes bibliographical referenes and index.
ISBN 978-1-118-92313-9 ((cloth : alk. paper)
1. Probabilities 2. Stochastic processes 3. Engineering-Statistical methods 4. Science-Statistical methods I, Title
QA273.K74 2015
519.2-dc23

Printed in the United States of America

10 9 8 7 6 5 4 3 2 1

2 2016

श्रीराम राम रामेति रमे रामे मनोरमे ।
सहस्रनाम तत्तुल्यं राम नाम वरानने ॥

Vishnu sahasranamam

Shree rāma rāma rāmēti
ramē rāmē manoramē
Sahasra nāmatathtulyam
rāmanāma varānanē

This book is respectfully dedicated to the
memory of my mother
Dharmambal Venkataraman
and the memory of my father
B. Venkataraman

மன்னனும் மாசறக்கற்றோனும் சீர்தூக்கின்
மன்னனிற்க்கற்றோன் சிறப்புடையோன்
மன்னர்க்குத் தன்தேசமல்லார்ச் சிறப்பில்லை
கற்றவனுக்குச் சென்றவிடெமல்லாம் சிறப்பு

Avvaiyar Tamil Poet

If a king and a well-learned person are balanced
A well-learned person weighs more than the king
Whereas the king is revered only in his country
A well-learned person is valued wherever he goes

On a well-learned person

CONTENTS

PREFACE FOR THE SECOND EDITION

In this second edition, additional sections have been incorporated to make it a more complete tome. In Chapter 4, a historical account of how Isaac Newton solved a binomial probability problem posed (1693) by the diarist Samuel Pepys when the field of probability was relatively unknown is described. In Chapter 10, Pearson correlation coefficient introduced earlier in the chapter has been expanded to include Spearman and Kendall correlation coefficients with their properties and usages discussed. In Chapter 14, convergences have been generalized. In Chapter 19 on estimation, two adaptive estimation techniques such as Recursive Least Squares and Least Mean Squares have been added. In Chapter 20 on random processes, additional topics such as Birth and Death Processes and Renewal Processes useful for analyzing queuing have been incorporated. Chapter 23 is a new chapter on Probability Modeling of Teletraffic Engineering written by my colleague Kavitha Chandra discussing various probability models in tele-traffic engineering.

This new edition now contains 455 carefully detailed figures and 377 representative examples, some with multiple solutions with every step explained clearly enhancing the clarity of presentation to yield a more comprehensive reference tome. It explains in detail the essential topics of applied mathematical functions to problems that engineers and researchers solve daily in the course of their work. The usual topics like set theory, combinatorics, random variables, discrete and continuous probability distribution functions, moments, and convergence of random variables, autocovariance and cross covariance functions, stationarity concepts, Wiener and Kalman filtering, and tomographic imaging of the first edition have all been retained. Graphical Fourier transform tables and a number of probability tables with accuracy up to nine decimal places are given in the seven appendices. A new eighth appendix on finding the roots of the probability generating function has also been added. The index has been carefully prepared to enhance the utility of this reference.

With these added material on theory and applications of probability, the second edition of Probability and Random Processes is a more comprehensive reference tome for practicing scientists and engineers in economics, science, and technology.

As in the first edition, the additional graphs in this edition were also created with Mathcad software. All algebraic calculations were also verified with Mathcad software. Mathcad is a registered trademark of Parametric Technology Corporation Inc., http://www.ptc.com/product/mathcad/

Acknowledgments

Since the publication of the first edition of this book, many other books on Probability have appeared and they have influenced me to a considerable extent in the choice of additional topics. I once again extend my scientific debt of gratitude to all these authors.

I acknowledge with great pleasure the support given to me by my colleague Kavitha Chandra who not only contributed a chapter but also helped me to a considerable extent in the preparation of the manuscript. My grateful thanks are due to Kari Capone who encouraged me to bring out a second edition and did the entire spade work to make this project

a reality. The courteous and thoughtful encouragement by Alex Castro during the final phases of the preparation of this manuscript is a pleasure for me to acknowledge. The efficiency with which Ramya Srinivasan, the production editor, went about the production job is indeed a pleasure to record. She also has the knack of smoothing ruffled feathers, which had kept me in an even keel. Finally, I also acknowledge with thanks the clean and efficient job performed by F. Pascal Raj, the assistant account manager and his team with their proactive suggestions.

The unequivocal support given to me by my wife Kamala has been a great source of comfort.

This is my third book with Wiley and I am always amazed at the way they can create a book with clockwork precision out of the rough manuscript.

VENKATARAMA KRISHNAN

Chelmsford, MA
March 2015

PREFACE FOR THE FIRST EDITION

Many good textbooks exist on probability and random processes written at the undergraduate level to the research level. However, there is no one handy and ready book that explains most of the essential topics, such as random variables and most of their frequently used discrete and continuous probability distribution functions; moments, transformation, and convergences of random variables; characteristic and generating functions; estimation theory and the associated orthogonality principle; vector random variables; random processes and their autocovariance and cross-covariance functions; stationarity concepts; and random processes through linear systems and the associated Wiener and Kalman filters. Engineering practitioners and students alike have to delve through several books to get the required formulas or tables either to complete a project or to finish a homework assignment. This book may alleviate this diffculty to some extent and provide access to a compendium of most distribution functions used by communication engineers, queuing theory specialists, signal processing engineers, biomedical engineers, and physicists. Probability tables with accuracy up to nine decimal places are given in the appendixes to enhance the utility of this book. A particular feature is the presentation of commonly occurring Fourier transforms where both the time and frequency functions are drawn to scale.

Most of the theory has been explained with figures drawn to scale. To understand the theory better, more than 300 examples are given with every step explained clearly. Following the adage that a figure is worth more than a thousand words, most of the examples are also illustrated with figures drawn to scale, resulting in more than 400 diagrams. This book will be of particular value to graduate and undergraduate students in electrical, computer, and civil engineering as well as students in physics and applied mathematics for solving homework assignments and projects. It will certainly be useful to communication and signal processing engineers, and computer scientists in an industrial setting. It will also serve as a good reference for research workers in biostatistics and financial market analysis.

The salient features of this book are

- Functional and statistical independence of random variables are explained.
- Ready reference to commonly occurring density and distribution functions and their means and variances is provided.
- A section on Benford's logarithmic law, which is used in detecting tax fraud, is included.
- More than 300 examples, many of them solved in different ways to illustrate the theory and various applications of probability, are presented.
- Most examples have been substantiated with graphs drawn to scale.
- More than 400 figures have been drawn to scale to improve the clarity of understanding the theory and examples.
- Bounds on the tails of Gaussian probability have been carefully developed.
- A chapter has been devoted to computer generation of random variates.
- Another chapter has been devoted to matrix algebra so that some estimation problems such as the Kalman filter can be cast in matrix framework.
- Estimation problems have been given considerable exposure.
- Random processes defined and classified.
- A section on martingale processes with examples has been included.

- Markov chains with interesting examples have been discussed.
- Wiener and Kalman filtering have been discussed with a number of examples.
- Important properties are summarized in tables.
- The final chapter is on applications of probability to tomographic imaging.
- The appendixes consist of probability tables with nine-place decimal accuracy for the most common probability distributions.
- An extremely useful feature are the Fourier transform tables with both time and frequency functions graphed carefully to scale.

After the introduction of functional independence in Section 1.2, the differences between functional and statistical independences are discussed in Section 2.3 and clarified in Example 2.3.2. The foundation for permutations and combinations are laid out in Chapter 3, followed by discrete distributions in Chapter 4 with an end-of-chapter summary of discrete distributions. Random variables are defined in Chapter 5, and most of the frequently occurring continuous distributions are explained in Chapters 6 and 7. Section 6.4 discusses the new bounds on Gaussian tails. A comprehensive table is given at the end of Chapter 7 for all the continuous distributions and densities. After discussion of conditional densities, joint densities, and moments in Chapters 8–10, characteristic and other allied functions are explained in Chapter 11 with a presentation of an end-of-chapter table of means and variances of all discrete and continuous distributions. Functions of random variables are discussed in Chapters 12 and 13 with numerous examples. Chapter 14 discusses the various bounds on probabilities along with some commonly occurring inequalities. Computer generation of random variates is presented in Chapter 15. Elements of matrix algebra along with vector and matrix differentiations are considered in Chapter 16. Vector random variables and diagonalization of covariance matrices and simultaneous diagonalization of two covariance matrices are taken up in Chapter 17. Estimation and allied hypothesis testing are discussed in Chapter 18. After random processes are explained in Chapter 19, they are carefully classified in Chapter 20, with Section 20.10 presenting martingale processes that find wide use in financial engineering. Chapter 21 discusses the effects of passing random processes through linear systems. A number of examples illustrate the basic ideas of Wiener and Kalman filters in Chapter 22. The final chapter, Chapter 23, presents a practical application of probability to tomographic imaging. The appendixes include probability tables up to nine decimal places for Gaussian, chi-square, Student-t, Poisson, and binomial distributions, and Fourier transform tables with graphs for time and frequency functions carefully drawn to scale.

This book is suitable for students and practicing engineers who need a quick reference to any probability distribution or table. It is also useful for self-study where a number of carefully solved examples illustrate the applications of probability. Almost every topic has solved examples. It can be used as an adjunct to any textbook on probability and may also be prescribed as a textbook with homework problems drawn from other sources.

During the more than four decades that I have been continuously teaching probability and random processes, many authors who have written excellent textbooks have influenced me to a considerable extent. Many of the ideas and concepts that are expanded in this book are the direct result of this influence. I owe a scientific debt of gratitude to all the authors who have contributed to this ubiquitous and exciting field of probability. Some of these authors are listed in the reference, and the list is by no means exhaustive.

Most of the graphs and all the probability tables in this book were created with Mathcad software. All algebraic calculations were verified with Mathcad software. Mathcad and Mathsoft are registered trademarks of Mathsoft Engineering and Education, Inc., http://www.mathcad. com.

While facing extreme personal diffculties in writing this book, the unequivocal support of my entire family have been a source of inspiration in finishing it.

I acknowledge with great pleasure the support given to me by the Wiley staff. George Telecki welcomed the idea of this book; Rachel Witmer did more than her share of keeping me in good humor with her cheerful disposition in spite of the ever-expanding deadlines; and Kellsee Chu who did an excellent job in managing the production of this book. Finally, the enthusiastic support given to me by the Series Editor Emmanuel Desurvire was very refreshing. This is my second book with Wiley, and the skills of their copyeditors and staff who transform highly mathematical manuscript into a finished book continue to amaze me.

I have revised this book several times and corrected errors. Nevertheless, I cannot claim that it is error-free since correcting errors in any book is a convergent process. I sincerely hope that readers will bring to my attention any errors of commission or omission that they may come across.

Chelmsford, Massachusetts VENKATARAMA KRISHNAN
March 2006

REFERENCES

1. Abromowitz, M., and I. A. Stegun, *Handbook of Mathematical Functions*, Dover Publications, 1972.
2. Akansu, A. N., and R. A. Haddad, *Multiresolution Signal Processing*, Academic Press, 1992.
3. Anderson, R. L., Distribution of the serial correlation coefficient, *Ann. Math. Stat.*, **13**, 1–13, 1942.
4. Aunon, J. I., and V. Chandrasekhar, *Introduction to Probability and Random Processes*, McGraw-Hill, 1997.
5. Benford, F., The law of anomalous numbers, *Proc. Am. Phil. Soc.*, **78**, 551–572, 1938.
6. Bharucha-Reid, A. T., *Elements of the Theory of Markov Processes and Their Applications*, Dover Publications, 1988.
7. Black, F., and M. Scholes, The pricing of options and corporate liabilities, *J. Polit. Econ.*, **81**, 637–654, 1973.
8. Börjesson, P. O., and C. E. W. Sundberg, Simple approximations of the error function Q(x) for communications applications, *IEEE Trans. Commun.*, **27**, 639–643, 1979.
9. Box, G. E. P., G. M. Jenkins, *and G. C. Reinsel, Time Series Analysis,* Prentice-Hall, 1994.
10. Chatfield, C., *The Analysis of Time Series: An Introduction, 2nd ed.,* Chapman & Hall, 1980.
11. Childers, D. G., *Probability and Random Processes*, Irwin, 1997.
12. Çinlar, E., Markov renewal theory, *Adv. Appl. Probab.*, **1**, 123–187, 1969.
13. Cooper, G. R., and C. D. McGillem, *Probabilistic Methods of Signal and System Analysis*, 3rd ed., Oxford University Press, 1999.
14. Cox, D. R., and H. D. Miller, *The Theory of Stochastic Processes*, Chapman & Hall/CRC, 1977.
15. Davenport, W. B., and W. L. Root, *An Introduction to the Theory of Random Signals and Noise*, IEEE Press, 1987.
16. David, F. N., *Games, Gods and Gambling*, Hafner Publishing Company, 1962.
17. Davis, J., *Performance Models for Correlated Traffic Data*, Ph. D. thesis, University of Massachusetts—Lowell, 2000.
18. Dhimani, J., S. Ahmad, and K. Gulia, Comparison between Adaptive filter Algorithms (LMS, NLMS and RLS), *Int. J. Sci. Eng. Technol. Res.*, **2**, 1100–1103, 2013.
19. Doob, J. L., *Stochastic Processes*, John Wiley & Sons, Inc., 1953.
20. Drenick, R., and R. Shahbender, Adaptive servomechanisms, *Trans. Am. Inst. Elect. Eng.*, **76**, 286–297, 1957.
21. Feller, W., *An Introduction to Probability Theory and Its Applications, 2nd ed., Vol. I*, John Wiley & Sons, Inc., 1968.
22. Fogiel, M., *Handbook of Mathematical, Scientific, and Engineering, Formulas, Tables, Functions, Graphs, Transforms*, Staff of Research and Education Association, 1988.
23. Fujisaki, M., G. Kallinapur, and H. Kunita, Stochastic differential equations for nonlinear filtering problem, *Osaka J. Math.*, **9**, 19–40, 1972.
24. Galton, F., Co-relations and their measurement, chiefly from anthropometric data, *Proc. R. Soc.*, **45**, 135–145, 1888.
25. Gantmacher, F., *The Theory of Matrices*, Chelsea Publishing, 1959.
26. Gardner, W. A., *Introduction to Random Processes, 2nd ed.,* McGraw-Hill, 1990.
27. Griffiths, D., A Pragmatic Approach to Spearman's Rank Correlation Coefficient, 1980, http://www.rsscseedu.org.uk/tsj/bts/griffiths/text.html. Available at December 17, 2014.
28. Grimmett, G., and D. Stirzaker, *Probability and Random Processes, 3rd ed.,* Oxford University Press, 2001.
29. Gross, D., and C. M. Harris, *Queuing Theory*, John Wiley & Sons, Inc., 1998.

30. Haykin, S., *Adaptive Filter Theory*, Prentice Hall, 2002.

31. Higgns, J. J., and S. Keller-McNulty, *Concepts in Probability and Stochastic Modeling*, Duxbury Press, 1995.

32. Hunter, J. J., On the Moments of Markov Renewal Processes, Institute of Statistics Mimeo Series No. 589, University of North Carolina.

33. Jenkins, G. M., and D. G. Watts, *Spectral Analysis and Its Applications*, Holden-Day, 1968.

34. Kallianpur, G., *Stochastic Filtering Theory*, Springer-Verlag, 1980.

35. Kalman, R. E., and R. S. Bucy, New results in linear filtering and prediction theory, *Trans. ASME J. Basic Eng. Ser. D*, **83**, 95–108, 1961.

36. Kamonpet, P., and V. Krishnan, *Modified EM Algorithm for Transmission Tomography, Internal Memo T94-1*, Center for Advanced Computation, University of Massachusetts Lowell, 1994.

37. Kamonpet, P., *Probabilistic Image Reconstruction from Noisy and Incomplete Projections*, Ph.D. thesis, University of Massachusetts—Lowell, 1995.

38. Kendall, M. G., A new measure of rank correlation, *Biometrika*, **30**, 81–93, 1938.

39. Kendall, M. G., The treatment of ties in ranking problems, *Biometrika*, **33**, 239–251, 1945.

40. Kleinrock, L., *Queueing Systems, Vol. I*. John Wiley & Sons, Inc., 1975.

41. Kleinrock, L., *Queuing Systems, Theory, Vol. II,* John Wiley & Sons, Inc., 1975.

42. Kleinrock, L., *Queuing Systems, Computer Applications, Vol. II,* John Wiley & Sons, Inc., 1976.

43. Krishnan, V., *Estimation of Autocovariance Functions,* Technical Report M-215U, Dynamics Research Corporation, 1974.

44. Krishnan, V., *Confidence Limits and Equivalence of Statistical Tests,* Technical Report M-256U, Dynamics Research Corporation, 1976.

45. Krishnan, V., *Distribution of Spectral Estimators,* Technical Report E-9334U, Dynamics Research Corporation, 1984.

46. Krishnan, V., *Nonlinear Filtering and Smoothing*, John Wiley & Sons, Inc., 1984.

47. Krishnan, V., *Linear Systems Properties*, CRC Press, 1998.

48. Krishnan, V., *Nonlinear Filtering and Smoothing*, Dover Publications, 2005.

49. Krishnan, V., *Probability and Random Processes*, Wiley-Interscience, 2006.

50. Krishnan, V., *Stochastic Processes: Classification, Encyclopedia of Statistical Science, Ed.* Miodrag Lovric, Springer, 2010,

51. Lam, T. C., *Superposition of Markov Renewal Processes and Their Applications,* Technical Report No. 90–21, Department of Industrial and Operations Engineering, University of Michigan, 1992.

52. Lange, K., Convergence of EM image reconstruction algorithms with Gibbs' smoothing, *IEEE Trans. Med. Imag.*, **9**, 439–446, 1990.

53. Leon-Garcia, A., *Probability and Random Processes for Electrical Engineering*, Addison-Wesley, 1993.

54. Levy, P., Semi-Markov systems having at most countable infinite possible states, *Proc. Intern. Congr. Math.*, **2**, 294, 1954; "Processus Semi-Markoviens," vol, 3, 416–426, 1954.

55. Lipster, R. S., and A. N. Shiryaev, *Statistics of Random Processes, Vols. I and II*, Springer-Verlag, 1977.

56. Macchi, O., *Adaptive Processing: The Least Mean Squares Approach with Applications in Transmission*, John Wiley & Sons, Ltd, 1995.

57. Meyer, P. A., A decomposition theorem for supermartingales, *Ill. J. Math.* **6**, 193–205, 1962.

58. Mood, A. M., and F. A. Graybill, *Introduction to the Theory of Statistics*, McGraw-Hill, 1963.

59. Mosteller, F., *Fifty Challenging Problems in Probability with Solutions, Problem 19*, Dover Publications, 1987.

60. Naidu, P. S. *Modern Spectrum Analysis of Time Series*, CRC Press, 1996.

61. Ostle, B., *Statistics in Research*, Iowa State University Press, 1963.

62. Papoulis, A., *Fourier Integral and Its Applications*, McGraw-Hill, 1962.

63. Papoulis, A., *Probability, Random Variables, and Stochastic Processes, 3rd ed.*, McGraw-Hill, 1991.

64. Pearson, K., Mathematical contributions to the theory of evolution-III. Regression, heredity and panmixia, *Philos. Trans. R. Soc.. A.*, **187**, 253–318, 1896.

65. Peebles Jr., P. Z., *Probability, Random Variables and Random Signal Principles, 4th ed.*, McGraw-Hill, 2001.

66. Plackett, R. L., Some theorems in least squares, *Biometrika*, **37**, 149–157, 1950.

67. Priestley, M. B., *Spectral Analysis and Time Series*, Academic Press, 1981.

68. Proakis, J. G., *Digital Communications*, McGraw-Hill, 1995.

69. Pyke, R., Markov renewal processes: Definitions and preliminary properties, *Ann. Math. Stat.*, **32**, 1231–1242, 1961.

70. Rice, S. O., *Mathematical analysis of random noise, in Selected Papers on Noise and Stochastic Processes, N. Wax, ed.*, Dover Publications, 1945.

71. Rome, H. J., and V. Krishnan, Causal probability model for evaluating future transoceanic airlane separations, *IEEE Trans. Aerosp. Electron. Syst.*, **26**, 804–817, 1990.

72. Ross, S. M., *A First Course in Probability, 6th ed.*, Prentice-Hall, 2002

73. Ross, S. M., *Introduction to Probability Models, 7th ed.*, Academic Press, 2000.

74. Ross, S. M., *Stochastic Processes, 2nd ed.*, John Wiley & Sons, Inc., 1996

75. Shanmugan, S., and A. M. Breiphol, *Random Signals, Detection, Estimation and Data Analysis*, John Wiley & Sons, Inc., 1988.

76. Shilov, G. E., *An Introduction to the Theory of Linear Spaces*, Prentice-Hall, 1961.

77. Sillitto, G. P., The distribution of Kendall's tau coefficient of rank correlation in rankings containing tie, *Biometrika*, **34**, 36–40, 1947.

78. Smith, W. L., Regenerative stochastic processes, *Proc. R. Soc A*, **232**, 6–31, 1955.

79. Srinath, M. D., P. K. Rajasekaran, and R. Viswanathan, *Introduction to Statistical Signal Processing with Applications*, Prentice-Hall, 1996.

80. Stark, H., and J. W. Woods, *Probability and Random Processes with Applications to Signal Processing,* 3rd ed., Prentice-Hall, 2002.

81. Stigler, S. M., Isaac Newton as a probabilist, *Stat. Sci.*, **21**, 400–403, 2006.

82. Taylor, H., and S. Karlin, *An Introduction to Stochastic Modeling, 3rd ed.*, Academic Press, 1998, pp 267–394.

83. Therrien, C. W., *Discrete Random Signals and Statistical Signal Processing*, Prentice-Hall, 1992.

84. Thomas, J. B., *An Introduction to Communication Theory and Systems*, Springer-Verlag, 1988.

85. Trivedi, K. S., *Probability and Statistics with Reliability, Queuing and Computer Science Applications*, Prentice-Hall, 1982.

86. Viniotis, Y., *Probability and Random Processes for Electrical Engineers*, McGraw-Hill, 1998.

87. Whitehead, A. N., and B. Russell, *Principia Mathematica*, Cambridge University Press, 1968.

88. Widrow, B., and S. D. Stearns, *Adaptive Signal Processing*, Prentice Hall, 1985

89. Widrow, B., and M. E. Hoff, Adaptive switching circuits, *Proc. WESCON Conv. Rec.*, **4**, 96–140, 1960.

90. Xu, L., *Probabilistic Reconstruction of Tomographic Images*, Ph.D. thesis, University of Massachusetts—Lowell, 1997.

91. Yates, R. D., and D. J. Goodman, *Probability and Stochastic Processes*, John Wiley & Sons, Inc., 1999.

WEB REFERENCES

92. David, F.N.: dspace.ucalgary.ca/bitstream/1880/41346/1/aih.pdf.

93. http://en.wikipedia.org/wiki/Samuel_Pepys-

94. http://mathworld.wolfram.com/Newton-PepysProblem.html

95. Noether, G. N.: http://www.teachingstatistics.co.uk/bts/noether text.html

96. Probability Handbook: http://www.itl.nist.gov/div898/handbook/eda/section3/eda36.htm

97. Stephen Stigler: http://arxiv.org/pdf/math/0701089.pdf

INDEX

Probability and Random Processes, Second Edition. Venkatarama Krishnan.

UPDATED WITH NEW CHAPTERS, FIGURES, AND APPENDICES TO COVER RECENT DEVELOPMENTS IN APPLIED MATHEMATICAL FUNCTIONS

This book examines the topics of applied mathematical functions to problems that engineers and researchers solve daily in the course of their work. The text covers set theory, combinatorics, random variables, discrete and continuous probability, distribution functions, convergence of random variables, computer generation of random variates, random processes and stationarity concepts with associated autocovariance and cross-covariance functions, estimation theory, and Wiener and Kalman filtering ending with two applications of probabilistic methods. Probability tables with nine decimal place accuracy and graphical Fourier transform tables are included for quick reference. The author facilitates understanding of probability concepts for both students and practitioners by presenting over 450 carefully detailed figures and illustrations and over 350 examples with every step explained clearly and some with multiple solutions.

Additional features of the second edition of *Probability and Random Processes* are as follows:

- Updated chapters with new sections on Newton–Pepys problem; Pearson, Spearman, and Kendal correlation coefficients; adaptive estimation techniques; birth and death processes; and renewal processes with generalizations
- A new chapter on Probability Modeling in Teletraffic Engineering written by Kavitha Chandra
- An appendix (Appendix H) examining the computation of the roots of discrete probability-generating functions.

With new material on theory and applications of probability, *Probability and Random Processes, Second Edition*, is a thorough and comprehensive reference for commonly occurring problems in probabilistic methods and their applications.

Venkatarama Krishnan, PhD, is Professor Emeritus in the Department of Electrical and Computer Engineering at the University of Massachusetts at Lowell having previously taught at Smith College, Indian Institute of Science, Polytechnic University in Brooklyn, University of Pennsylvania, Villanova University and Princeton University. He has served as a consultant to the Dynamics Research Corporation, the U.S. Department of Transportation, and Bell Laboratories. Dr. Krishnan's research includes estimation of steady-state queue distribution, tomographic imaging, aerospace, control, communications, and stochastic systems. Dr. Krishnan is a senior member of the IEEE and listed in Who is Who in America.

Cover Design: Venkatarama Krishnan
Cover Image: Covey of beta densities

ISBN 978-1-118-92313-9

WILEY

Also available as an e-book