

S E C O N D E D I T I O N



Probability and Random Processes

Venkatarama Krishnan

with contribution from
Kavitha Chandra

WILEY

SECOND EDITION

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Random Processes*

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PREVIEW

**PROBABILITY AND RANDOM
PROCESSES**

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SECOND EDITION

VENKATARAMA KRISHNAN

Professor Emeritus of Electrical and Computer Engineering
Center for Advanced Computation and Telecommunications
University of Massachusetts Lowell

With Contribution from

KAVITHA CHANDRA

Professor of Electrical and Computer Engineering
Center for Advanced Computation and Telecommunications
University of Massachusetts Lowell

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श्रीराम राम रामेति रमे रामे मनोरमे ।
सहस्रनाम तत्तुल्यं राम नाम वरानने ॥

Vishnu sahasranamam

Shree rāma rāma rāmēti
ramē rāmē manoramē
Sahasra nāmatathtulyam
rāmanāma varānanē



This book is respectfully dedicated to the
memory of my mother
Dharmambal Venkataraman
and the memory of my father
B. Venkataraman

மன்னனும் மாசறக்கற்றோனும் சீர்தூக்கின்
மன்னனிற்க்கற்றோன் சிறப்புடையோன்
மன்னர்க்குத் தன்தேசமல்லார்ச் சிறப்பில்லை
கற்றவனுக்குச் சென்றவிடெமல்லாம் சிறப்பு

Avvaiyar Tamil Poet

If a king and a well-learned person are balanced
A well-learned person weighs more than the king
Whereas the king is revered only in his country
A well-learned person is valued wherever he goes

On a well-learned person

CONTENTS

Preface for the Second Edition	xii
Preface for the First Edition	xiv
1 Sets, Fields, and Events	1
1.1 Set Definitions, 1	
1.2 Set Operations, 2	
1.3 Set Algebras, Fields, and Events, 5	
2 Probability Space and Axioms	7
2.1 Probability Space, 7	
2.2 Conditional Probability, 9	
2.3 Independence, 11	
2.4 Total Probability and Bayes' Theorem, 12	
3 Basic Combinatorics	16
3.1 Basic Counting Principles, 16	
3.2 Permutations, 16	
3.3 Combinations, 18	
4 Discrete Distributions	23
4.1 Bernoulli Trials, 23	
4.2 Binomial Distribution, 23	
4.3 Multinomial Distribution, 26	
4.4 Geometric Distribution, 26	
4.5 Negative Binomial Distribution, 27	
4.6 Hypergeometric Distribution, 28	
4.7 Poisson Distribution, 30	
4.8 Newton–Pepys Problem and its Extensions, 33	
4.9 Logarithmic Distribution, 40	
4.9.1 Finite Law (Benford's Law), 40	
4.9.2 Infinite Law, 43	
4.10 Summary of Discrete Distributions, 44	

5	Random Variables	45
5.1	Definition of Random Variables, 45	
5.2	Determination of Distribution and Density Functions, 46	
5.3	Properties of Distribution and Density Functions, 50	
5.4	Distribution Functions from Density Functions, 51	
6	Continuous Random Variables and Basic Distributions	54
6.1	Introduction, 54	
6.2	Uniform Distribution, 54	
6.3	Exponential Distribution, 55	
6.4	Normal or Gaussian Distribution, 57	
7	Other Continuous Distributions	63
7.1	Introduction, 63	
7.2	Triangular Distribution, 63	
7.3	Laplace Distribution, 63	
7.4	Erlang Distribution, 64	
7.5	Gamma Distribution, 65	
7.6	Weibull Distribution, 66	
7.7	Chi-Square Distribution, 67	
7.8	Chi and Other Allied Distributions, 68	
7.9	Student-t Density, 71	
7.10	Snedecor F Distribution, 72	
7.11	Lognormal Distribution, 72	
7.12	Beta Distribution, 73	
7.13	Cauchy Distribution, 74	
7.14	Pareto Distribution, 75	
7.15	Gibbs Distribution, 75	
7.16	Mixed Distributions, 75	
7.17	Summary of Distributions of Continuous Random Variables, 76	
8	Conditional Densities and Distributions	78
8.1	Conditional Distribution and Density for $P\{A\} \neq 0$, 78, 82	
	Conditional Distribution and Density for $P\{A\} = 0$, 80, 83	
	Total Probability and Bayes' Theorem for Densities, 83	
9	Joint Densities and Distributions	85
9.1	Joint Discrete Distribution Functions, 85	
9.2	Joint Continuous Distribution Functions, 86	
9.3	Bivariate Gaussian Distributions, 90	
10	Moments and Conditional Moments	91
10.1	Expectations, 91	
10.2	Variance, 92	
10.3	Means and Variances of Some Distributions, 93	
10.4	Higher-Order Moments, 94	
10.5	Correlation and Partial Correlation Coefficients, 95	
	10.5.1 Correlation Coefficients, 95	
	10.5.2 Partial Correlation Coefficients, 106	

11	Characteristic Functions and Generating Functions	108
	11.1 Characteristic Functions, 108	
	11.2 Examples of Characteristic Functions, 109	
	11.3 Generating Functions, 111	
	11.4 Examples of Generating Functions, 112	
	11.5 Moment Generating Functions, 113	
	11.6 Cumulant Generating Functions, 115	
	11.7 Table of Means and Variances, 116	
12	Functions of a Single Random Variable	118
	12.1 Random Variable $g(X)$, 118	
	12.2 Distribution of $Y=g(X)$, 119	
	12.3 Direct Determination of Density $f_Y(y)$ from $f_X(x)$, 129	
	12.4 Inverse Problem: Finding $g(X)$ given $f_X(x)$ and $f_Y(y)$, 132	
	12.5 Moments of a Function of a Random Variable, 133	
13	Functions of Multiple Random Variables	135
	13.1 Function of Two Random Variables, $Z = g(X,Y)$, 135	
	13.2 Two Functions of Two Random Variables, $Z = g(X,Y)$, $W = h(X,Y)$, 143	
	13.3 Direct Determination of Joint Density $f_{ZW}(z,w)$ from $f_{XY}(x,y)$, 146	
	13.4 Solving $Z = g(X,Y)$ Using an Auxiliary Random Variable, 150	
	13.5 Multiple Functions of Random Variables, 153	
14	Inequalities, Convergences, and Limit Theorems	155
	14.1 Degenerate Random Variables, 155	
	14.2 Chebyshev and Allied Inequalities, 155	
	14.3 Markov Inequality, 158	
	14.4 Chernoff Bound, 159	
	14.5 Cauchy–Schwartz Inequality, 160	
	14.6 Jensen’s Inequality, 162	
	14.7 Convergence Concepts, 163	
	14.8 Limit Theorems, 165	
15	Computer Methods for Generating Random Variates	169
	15.1 Uniform-Distribution Random Variates, 169	
	15.2 Histograms, 170	
	15.3 Inverse Transformation Techniques, 172	
	15.4 Convolution Techniques, 178	
	15.5 Acceptance–Rejection Techniques, 178	
16	Elements of Matrix Algebra	181
	16.1 Basic Theory of Matrices, 181	
	16.2 Eigenvalues and Eigenvectors of Matrices, 186	
	16.3 Vector and Matrix Differentiation, 190	
	16.4 Block Matrices, 194	
17	Random Vectors and Mean-Square Estimation	196
	17.1 Distributions and Densities, 196	

- 17.2 Moments of Random Vectors, 200
- 17.3 Vector Gaussian Random Variables, 204
- 17.4 Diagonalization of Covariance Matrices, 207
- 17.5 Simultaneous Diagonalization of Covariance Matrices, 209
- 17.6 Linear Estimation of Vector Variables, 210

18 Estimation Theory 212

- 18.1 Criteria of Estimators, 212
- 18.2 Estimation of Random Variables, 213
- 18.3 Estimation of Parameters (Point Estimation), 218
- 18.4 Interval Estimation (Confidence Intervals), 225
- 18.5 Hypothesis Testing (Binary), 231
- 18.6 Bayesian Estimation, 238

19 Random Processes 250

- 19.1 Basic Definitions, 250
- 19.2 Stationary Random Processes, 258
- 19.3 Ergodic Processes, 269
- 19.4 Estimation of Parameters of Random Processes, 273
 - 19.4.1 Continuous-Time Processes, 273
 - 19.4.2 Discrete-Time Processes, 280
- 19.5 Power Spectral Density, 287
 - 19.5.1 Continuous Time, 287
 - 19.5.2 Discrete Time, 294
- 19.6 Adaptive Estimation, 298

20 Classification of Random Processes 320

- 20.1 Specifications of Random Processes, 320
 - 20.1.1 Discrete-State Discrete-Time (DSDT) Process, 320
 - 20.1.2 Discrete-State Continuous-Time (DSCT) Process, 320
 - 20.1.3 Continuous-State Discrete-Time (CSDT) Process, 320
 - 20.1.4 Continuous-State Continuous-Time (CSCT) Process, 320
- 20.2 Poisson Process, 321
- 20.3 Binomial Process, 329
- 20.4 Independent Increment Process, 330
- 20.5 Random-Walk Process, 333
- 20.6 Gaussian Process, 338
- 20.7 Wiener Process (Brownian Motion), 340
- 20.8 Markov Process, 342
- 20.9 Markov Chains, 347
- 20.10 Birth and Death Processes, 357
- 20.11 Renewal Processes and Generalizations, 366
- 20.12 Martingale Process, 370
- 20.13 Periodic Random Process, 374
- 20.14 Aperiodic Random Process (Karhunen–Loeve Expansion), 377

21 Random Processes and Linear Systems 383

- 21.1 Review of Linear Systems, 383
- 21.2 Random Processes through Linear Systems, 385
- 21.3 Linear Filters, 393
- 21.4 Bandpass Stationary Random Processes, 401

22	Wiener and Kalman Filters	413
	22.1 Review of Orthogonality Principle, 413	
	22.2 Wiener Filtering, 414	
	22.3 Discrete Kalman Filter, 425	
	22.4 Continuous Kalman Filter, 433	
23	Probability Modeling in Tele-Traffic Engineering	437
	(Kavitha Chandra)	
	23.1 Introduction, 437	
	23.2 Teletraffic Models, 437	
	23.3 Blocking Systems, 438	
	23.4 State Probabilities for Systems with Delays, 440	
	23.5 Waiting-Time Distribution for M/M/c/∞ Systems, 441	
	23.6 State Probabilities for M/D/c Systems, 443	
	23.7 Waiting-Time distribution for M/D/c/∞ System, 446	
	23.8 Comparison of M/M/c and M/D/c, 448	
	References, 451	
24	Probabilistic Methods in Transmission Tomography	452
	24.1 Introduction, 452	
	24.2 Stochastic Model, 453	
	24.3 Stochastic Estimation Algorithm, 455	
	24.4 Prior Distribution $P\{\mathbf{M}\}$, 457	
	24.5 Computer Simulation, 458	
	24.6 Results and Conclusions, 460	
	24.7 Discussion of Results, 462	
	References, 462	
APPENDICES		
A	A Fourier Transform Tables	463
B	Cumulative Gaussian Tables	467
C	Inverse Cumulative Gaussian Tables	472
D	Inverse Chi-Square Tables	474
E	Inverse Student-<i>t</i> TABLES	481
F	Cumulative Poisson Distribution	484
G	Cumulative Binomial Distribution	488
H	Computation of Roots of $D(z) = 0$	494
	References	495
	Index	498

PREFACE FOR THE SECOND EDITION

In this second edition, additional sections have been incorporated to make it a more complete tome. In Chapter 4, a historical account of how Isaac Newton solved a binomial probability problem posed (1693) by the diarist Samuel Pepys when the field of probability was relatively unknown is described. In Chapter 10, Pearson correlation coefficient introduced earlier in the chapter has been expanded to include Spearman and Kendall correlation coefficients with their properties and usages discussed. In Chapter 14, convergences have been generalized. In Chapter 19 on estimation, two adaptive estimation techniques such as Recursive Least Squares and Least Mean Squares have been added. In Chapter 20 on random processes, additional topics such as Birth and Death Processes and Renewal Processes useful for analyzing queuing have been incorporated. Chapter 23 is a new chapter on Probability Modeling of Teletraffic Engineering written by my colleague Kavitha Chandra discussing various probability models in tele-traffic engineering.

This new edition now contains 455 carefully detailed figures and 377 representative examples, some with multiple solutions with every step explained clearly enhancing the clarity of presentation to yield a more comprehensive reference tome. It explains in detail the essential topics of applied mathematical functions to problems that engineers and researchers solve daily in the course of their work. The usual topics like set theory, combinatorics, random variables, discrete and continuous probability distribution functions, moments, and convergence of random variables, autocovariance and cross covariance functions, stationarity concepts, Wiener and Kalman filtering, and tomographic imaging of the first edition

have all been retained. Graphical Fourier transform tables and a number of probability tables with accuracy up to nine decimal places are given in the seven appendices. A new eighth appendix on finding the roots of the probability generating function has also been added. The index has been carefully prepared to enhance the utility of this reference.

With these added material on theory and applications of probability, the second edition of Probability and Random Processes is a more comprehensive reference tome for practicing scientists and engineers in economics, science, and technology.

As in the first edition, the additional graphs in this edition were also created with Mathcad software. All algebraic calculations were also verified with Mathcad software. Mathcad is a registered trademark of Parametric Technology Corporation Inc., <http://www.ptc.com/product/mathcad/>

Acknowledgments

Since the publication of the first edition of this book, many other books on Probability have appeared and they have influenced me to a considerable extent in the choice of additional topics. I once again extend my scientific debt of gratitude to all these authors.

I acknowledge with great pleasure the support given to me by my colleague Kavitha Chandra who not only contributed a chapter but also helped me to a considerable extent in the preparation of the manuscript. My grateful thanks are due to Kari Capone who encouraged me to bring out a second edition and did the entire spade work to make this project

a reality. The courteous and thoughtful encouragement by Alex Castro during the final phases of the preparation of this manuscript is a pleasure for me to acknowledge. The efficiency with which Ramya Srinivasan, the production editor, went about the production job is indeed a pleasure to record. She also has the knack of smoothing ruffled feathers, which had kept me in an even keel. Finally, I also acknowledge with thanks the clean and efficient job performed by F. Pascal Raj, the assistant account manager and his team with their proactive suggestions.

The unequivocal support given to me by my wife Kamala has been a great source of comfort.

This is my third book with Wiley and I am always amazed at the way they can create a book with clockwork precision out of the rough manuscript.

VENKATARAMA KRISHNAN
Chelmsford, MA
March 2015

PREFACE FOR THE FIRST EDITION

Many good textbooks exist on probability and random processes written at the undergraduate level to the research level. However, there is no one handy and ready book that explains most of the essential topics, such as random variables and most of their frequently used discrete and continuous probability distribution functions; moments, transformation, and convergences of random variables; characteristic and generating functions; estimation theory and the associated orthogonality principle; vector random variables; random processes and their autocovariance and cross-covariance functions; stationarity concepts; and random processes through linear systems and the associated Wiener and Kalman filters. Engineering practitioners and students alike have to delve through several books to get the required formulas or tables either to complete a project or to finish a homework assignment. This book may alleviate this difficulty to some extent and provide access to a compendium of most distribution functions used by communication engineers, queuing theory specialists, signal processing engineers, biomedical engineers, and physicists. Probability tables with accuracy up to nine decimal places are given in the appendixes to enhance the utility of this book. A particular feature is the presentation of commonly occurring Fourier transforms where both the time and frequency functions are drawn to scale.

Most of the theory has been explained with figures drawn to scale. To understand the theory better, more than 300 examples are given with every step explained clearly. Following the adage that a figure is worth more than a thousand words, most of the examples are also illustrated with figures drawn to scale, resulting in more than 400 diagrams. This book will be of particular value to graduate and undergraduate students in electrical, computer, and civil engineering as well as students in physics and applied mathematics for solving homework assignments and projects. It will certainly be

useful to communication and signal processing engineers, and computer scientists in an industrial setting. It will also serve as a good reference for research workers in biostatistics and financial market analysis.

The salient features of this book are

- Functional and statistical independence of random variables are explained.
- Ready reference to commonly occurring density and distribution functions and their means and variances is provided.
- A section on Benford's logarithmic law, which is used in detecting tax fraud, is included.
- More than 300 examples, many of them solved in different ways to illustrate the theory and various applications of probability, are presented.
- Most examples have been substantiated with graphs drawn to scale.
- More than 400 figures have been drawn to scale to improve the clarity of understanding the theory and examples.
- Bounds on the tails of Gaussian probability have been carefully developed.
- A chapter has been devoted to computer generation of random variates.
- Another chapter has been devoted to matrix algebra so that some estimation problems such as the Kalman filter can be cast in matrix framework.
- Estimation problems have been given considerable exposure.
- Random processes defined and classified.
- A section on martingale processes with examples has been included.

- Markov chains with interesting examples have been discussed.
- Wiener and Kalman filtering have been discussed with a number of examples.
- Important properties are summarized in tables.
- The final chapter is on applications of probability to tomographic imaging.
- The appendixes consist of probability tables with nine-place decimal accuracy for the most common probability distributions.
- An extremely useful feature are the Fourier transform tables with both time and frequency functions graphed carefully to scale.

After the introduction of functional independence in Section 1.2, the differences between functional and statistical independences are discussed in Section 2.3 and clarified in Example 2.3.2. The foundation for permutations and combinations are laid out in Chapter 3, followed by discrete distributions in Chapter 4 with an end-of-chapter summary of discrete distributions. Random variables are defined in Chapter 5, and most of the frequently occurring continuous distributions are explained in Chapters 6 and 7. Section 6.4 discusses the new bounds on Gaussian tails. A comprehensive table is given at the end of Chapter 7 for all the continuous distributions and densities. After discussion of conditional densities, joint densities, and moments in Chapters 8–10, characteristic and other allied functions are explained in Chapter 11 with a presentation of an end-of-chapter table of means and variances of all discrete and continuous distributions. Functions of random variables are discussed in Chapters 12 and 13 with numerous examples. Chapter 14 discusses the various bounds on probabilities along with some commonly occurring inequalities. Computer generation of random variates is presented in Chapter 15. Elements of matrix algebra along with vector and matrix differentiations are considered in Chapter 16. Vector random variables and diagonalization of covariance matrices and simultaneous diagonalization of two covariance matrices are taken up in Chapter 17. Estimation and allied hypothesis testing are discussed in Chapter 18. After random processes are explained in Chapter 19, they are carefully classified in Chapter 20, with Section 20.10 presenting martingale processes that find wide use in financial engineering. Chapter 21 discusses the effects of passing random processes through linear systems. A number of examples illustrate the basic ideas of Wiener and Kalman filters in Chapter 22. The final chapter, Chapter 23, presents a practical application of

probability to tomographic imaging. The appendixes include probability tables up to nine decimal places for Gaussian, chi-square, Student-t, Poisson, and binomial distributions, and Fourier transform tables with graphs for time and frequency functions carefully drawn to scale.

This book is suitable for students and practicing engineers who need a quick reference to any probability distribution or table. It is also useful for self-study where a number of carefully solved examples illustrate the applications of probability. Almost every topic has solved examples. It can be used as an adjunct to any textbook on probability and may also be prescribed as a textbook with homework problems drawn from other sources.

During the more than four decades that I have been continuously teaching probability and random processes, many authors who have written excellent textbooks have influenced me to a considerable extent. Many of the ideas and concepts that are expanded in this book are the direct result of this influence. I owe a scientific debt of gratitude to all the authors who have contributed to this ubiquitous and exciting field of probability. Some of these authors are listed in the reference, and the list is by no means exhaustive.

Most of the graphs and all the probability tables in this book were created with Mathcad software. All algebraic calculations were verified with Mathcad software. Mathcad and Mathsoft are registered trademarks of Mathsoft Engineering and Education, Inc., <http://www.mathcad.com>.

While facing extreme personal difficulties in writing this book, the unequivocal support of my entire family have been a source of inspiration in finishing it.

I acknowledge with great pleasure the support given to me by the Wiley staff. George Telecki welcomed the idea of this book; Rachel Witmer did more than her share of keeping me in good humor with her cheerful disposition in spite of the ever-expanding deadlines; and Kellsee Chu who did an excellent job in managing the production of this book. Finally, the enthusiastic support given to me by the Series Editor Emmanuel Desurvire was very refreshing. This is my second book with Wiley, and the skills of their copyeditors and staff who transform highly mathematical manuscript into a finished book continue to amaze me.

I have revised this book several times and corrected errors. Nevertheless, I cannot claim that it is error-free since correcting errors in any book is a convergent process. I sincerely hope that readers will bring to my attention any errors of commission or omission that they may come across.

Chelmsford, Massachusetts VENKATARAMA KRISHNAN
March 2006

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INDEX

- adaptive coefficients, 193
- adaptive estimation, 298
- aperiodic random processes, 377
 - discrete Karhunen—Loeve (K—L) expansion, 381
 - Karhunen—Loeve (K—L) expansion, 378
 - eigenfunctions, 378
 - eigenvalues, 378
 - nonstationary process, 378
- arcsine law, 127, 128
- AR(1) process, 339
- asymptotic probabilities, 447
- attenuation coefficient, 453
- autoregressive process, 269
- average queue length, 442
- average waiting time, 442, 447, 449

- bandpass signals, 401
 - analytic signal representation, 401
 - complex envelope, 403
 - natural envelope, 401
 - lower side band, 401
 - upper side band, 401
 - quadrature representation, 401
- Bayesian estimation, 238
 - discrete probability, 245
 - MAP criterion, 244
 - ML criterion, 244
- Bayes' theorem, 12, 454
- Bayes' theorem for densities, 83

- Bernoulli, 23
 - distribution, 23
 - trials, 23
 - independent identically distributed, 23
- Bernoulli process, 329
 - modified, 332
- Bessel function of ν th order, 70
 - zero order Bessel function, 141, 410
- biased dice, 36
- bilinear form, 191
- binomial coefficient, 18
 - Pascal's identity, 18
 - triangle, 19
 - properties, 18
 - sum, 19
 - Vandermionde's identity, 19
- binomial distribution, 366
- binomial distribution solution, 35
- binomial process, 329, 331
- birth and death process, 357
 - continuous time Markov chain, 357
 - differential equations formulation, 358
 - infinitesimal generator matrix, 358
 - steady state solution, 359
- birthdays problem, 17, 32
- bivariate Gaussian, 96
 - conditional expectation, 96
 - conditional variance, 96
 - correlation coefficient, 96
 - covariance, 96
 - joint density, 96
- bivariate Gaussian distribution, 90, 96

- bivariate stochastic process, 369
- blocked calls, 438
 - cleared system (BCC), 438, 440
 - delayed system (BCD), 438, 440
 - probability, 438
- blocking probability, 439, 441, 442, 450
- block matrices, 194
 - properties, 194
- Bose—Einstein statistics, 20
- bound, 155
 - Chebyshev, 156
 - alternate forms, 156
 - geometric derivation, 156
 - Chemoff, 159
- Brownian motion, 321, 340 *see also* Wiener process

- Catalan numbers, 20
- Cauchy, 74
 - density, 149
 - distribution, 74, 142, 143, 146
- Cauchy—Riemann conditions, 192
- Cayley—Hamilton theorem, 187
- central limit theorem, 166
- characteristic functions, 108, 138, 139
 - examples, 109
 - continuous random variables, 110
 - discrete random variables, 109
 - existence, 108
 - joint characteristic functions, 109
 - moment generating properties, 108
- chi-squared test, 171
- combinations, 18
 - sampling without replacement, 18
 - sampling with replacement, 20
- comparison of Gaussian and Poisson approximations, 62
- computer methods for random variates, 169
 - acceptance-rejection, 178
 - algorithm, 179
 - beta density, 179
 - comparison function, 179
 - convolution techniques, 178
 - Erlang distribution, 178
 - triangular distribution, 178
 - inverse transformation, 171
- conditional densities, 78
- conditional distributions, 78
 - properties, 78
- conditional distributions and densities, 78, 197
 - clarity, 198
 - $P(A) = 0$, 80
 - $P(A) > 0$, 78
- conditional expectations, 368, 370
- confidence coefficient, 218, 226
- confidence interval, 225, 226
- coefficients of simple linear regression, 230
- unknown mean of population, 226
 - known variance, 226
 - unknown variance, 227
 - unknown variance with unknown mean, 227
- confidence region, 230
 - simple linear regression line, 230
- congruential generator, 169
 - linear, 169
 - mixed, 169
 - multiplicative, 169
- continuous distributions, 54, 63
 - beta distribution, 73
 - Cauchy distribution, 74
 - chi distribution, 68
 - Maxwell density, 69
 - Nakagami density, 70
 - Rayleigh density, 68
 - Rice's density, 69
 - chi-square, 67
 - double Laplace distribution, 75
 - Erlang distribution, 64
 - exponential distribution, 55
 - hazard rate, 56
 - instantaneous, 56
 - Poisson arrival process, 55
 - gamma distribution, 65
 - reproductive property, 66
 - Gaussian distribution, 57
 - Gibbs distribution, 75
 - potential function, 75
 - Laplace distribution, 63
 - double Laplace distribution, 75
 - lognormal distribution, 72
 - Pareto distribution, 75
 - Snedecor F-distribution, 72
 - student-t density, 71
 - summary of continuous distributions, 76
 - triangular, 63
 - uniform distribution, 54
 - Weibull, 66
- continuous Kalman filter, 433
 - assumptions, 434
 - matrix Riccati equation, 434
- continuous random variables, 94
 - means and variances, 93
 - exponential distribution, 94
 - Gaussian distribution, 94
 - convergence, 163, 307
- almost sure, 164
- Cauchy criterion, 164
 - in distribution, 165
 - in mean, 165
 - in probability, 164
- mean square, 164
 - pointwise, 164
 - properties, 165
- quadratic mean, 164
- rth mean, 165
- convergences of RLS and LMS algorithms, 307
- conditions for mean square error, 316, 318
 - in the mean, 307, 313, 314
 - mean square error, 316, 318
 - convex function, 162

- correlation matrix, 300
 - inverse correlation matrix, 302
- counting, 16
 - addition rule, 16
 - multiplication rule, 16
- counting process, 366
- covariance matrices, 203
 - diagonalization, 207
 - diagonalization to an identity matrix, 207
 - principles, 207
 - simultaneous diagonalization, 209
 - summary of procedures, 210
- covariance of ACF estimators, 276
- craps game, 10
- criterion function, 301
- cumulant generating functions, 115
- cumulants, 115
 - Gaussian distribution, 115
 - joint cumulants, 116
 - Poisson distribution, 115
- customers
 - arrival rates, 437, 442
 - cyclical order, 446
 - departure rates, 437, 438, 440
 - queue, 448
- cyclostationary process, 375

- De Moivre—Laplace limit theorem, 38, 40, 61
- density function, 46
- differentiation, 190
 - complex variable, 192
 - Cauchy—Riemann conditions, 192
 - partial derivative, 192
 - regular point, 192
 - singular point, 192
 - total derivative, 192
 - complex vectors, 193
 - partial differentiation, 192
 - real matrices, 194
 - real vectors, 190
- direct determination of density, 129
 - conditions, 129
 - steps, 129
- direct determination of joint density, 146
- discrete distributions, 23
 - Bernoulli, 23
 - Benford, 40
 - binomial, 23
 - approximation, 31
 - geometric, 26
 - hypergeometric, 28
 - logarithmic, 40
 - finite law, 40
 - first significant digit, 40
 - second significant digit, 42
 - infinite law, 43
 - multinomial, 26
 - negative binomial, 27
 - Pascal distribution, 27
 - Poisson, 30
 - approximation to binomial, 31
 - summary, 44
- discrete Kalman filter, 425
 - across observations, 426, 427
 - algorithm, 430
 - assumptions, 427
 - between observations, 426, 427
 - comments, 429
 - covariance propagation, 430
 - Kalman gain, 431
 - scalar case, 432
 - timing diagram, 426, 429
- discrete random variables, 93
 - means and variances, 93
 - binomial distribution, 93
 - Poisson distribution, 94
 - variance, 278
- discrete time processes, 280
 - Anderson's theorem, 286
 - estimation of autocorrelation functions, 285
 - estimation of covariance of NACF, 286
 - estimation of NACF, 286
 - large-lag approximations, 286
 - variance of autocovariance estimators, 284
 - white noise, 287
- discrete time stationary processes, 268
 - moments, 268
- distribution and density functions, 46
 - properties, 50
 - cumulative distribution function $F_x(x)$, 50
 - probability density function $f_x(x)$, 51
 - probability mass function, 51
- distribution function, 45
 - from density, 51
- distribution of $Y = g(X)$, 119
 - steps for determination, 119

- eigenvalues of matrices, 186
 - characteristic polynomial, 186
- eigenvectors of matrices, 187
 - normalization, 188
- emission tomography, 452
- epochs, 366
- ergodic processes, 269
 - correlation ergodic, 269
 - mean-ergodic, 269
 - power ergodic, 271
 - time averages, 271
- Erlang distribution, 367
- Erlang loss formul B, C, 439
- error surface, 311
- estimated regression coefficients, 222
 - covariance, 223
 - from data, 222
- estimated regression line, 223
 - variance, 223
- estimation criteria, 212

- consistent, 213
- efficient, 213
 - Rao—Cramer bound, 213
- log-likelihood, 213
- maximum likelihood, 213
- mean square error, 212
- minimum variance, 212
- unbiased, 212
- estimation error, 299
- estimation of a random variable, 213
 - by a constant, 213
 - by a function of another random variable, 213
- estimation theory, 212
 - likelihood function, 240
 - likelihood ratio, 240
 - linear estimation of vector variables, 210
 - overdetermined system, 211
 - underdetermined system, 211
 - maximum a posteriori probability (MAP), 240
 - minimization of average probability of error, 241
 - maximum likelihood ML, 241
 - properties of ML estimators, 242
 - parameter estimation, 218
 - Bayesian estimation, 238
 - covariance, 221
 - higher order moments, 222
 - interval estimation, 218, 225
 - mean, 218, 226
 - unbiased, 218, 226
 - minimum mean square estimation, 219
 - point estimation, 218
 - variance, 220
 - standard error, 220
 - statistic, 218
- excess mean square error, 310, 316
- expectation maximization algorithm, 456
 - one step late, 456
 - RMS error, 460
- fair coins toss, 37
- field, 5, 7
 - Boolean field, 5
 - Borel cv-field, 6
- filter coefficients, 304, 310
- functions of a single random variable, 118
 - higher order moments, 133
 - moments, 133
 - random variable $g(X)$, 118
 - conditions, 118
- function of two random variables, 135
 - conditions, 135
 - image, 135
 - inverse image, 135
 - steps for determining $FZ(z)$, 136
- gambler's ruin, 337
- gamma function, 65
- incomplete, 65
- Gaussian approximation to binomial distribution, 61
 - Demoivre—Laplace limit theorem, 61
- Gaussian moment factoring theorem, 315
- Gaussian probability, 57
 - density function, 57
 - distribution function, 57
 - mean, 57
 - standard, 57
 - properties, 59
 - tails function $g(x)$, 59
 - lower bound, 60
 - properties, 60
 - upper bound, 59
 - variance, 57
- Gaussian process, 338
 - properties, 339
- generating functions, 111
 - examples, 112
 - binomial distribution, 112
 - geometric distribution, 113
 - Poisson distribution, 112
 - moment generating properties, 112
- geometric series, 308
- Gibbs' distribution, 75
 - potential function, 457
 - Incosh, 457
 - sigmoid, 457
- Gibbs' prior, 452
- grade of service, 437, 439, 440, 442
- higher order moments, 94
 - cross moments, 95
- Hilbert transform, 401
 - table, 403
- histograms, 170
- holding time, 437, 438, 441, 448
- hypothesis testing, 231
 - acceptance region, 231
 - alternate hypothesis, 231
 - binary, 231
 - composite hypothesis, 232
 - cost for errors added, 247
 - mean p ,-known variance cv^2 , 234
 - mean p ,-unknown variance cv^2 , 235
 - ML rule, 242
 - null hypothesis, 231
 - number of samples n for given a and b , 237
 - operating characteristic, 233
 - power of the test, 233
 - procedure for testing, 234
 - rejection region, 231
 - significance testing, 231, 232
 - simple hypothesis, 232
 - statistic, 231
- image, 45
- independence, 3
 - functional, 3, 11

- statistical, 3, 11
- independent increment process, 330
 - stationary independent increment, 330
- indicator function, 134, 156
 - properties, 134
- inequalities, 155
 - Cauchy—Schwartz, 160
 - frequency domain, 162
 - Chebyshev, 155
 - generalized Chebyshev, 157
 - Jensen, 162
 - Markov, 158
- innovations process, 426
- integrating factor, 361, 362, 365
- interarrival time, 366-8
- interval estimation, 225
- inverse Hilbert transform, 402
- inverse image, 45
- inverse problem $g(x)$ from $f(x)$ and $f(y)$, 131
- inverse transformation, 171
 - continuous distributions, 174
 - exponential distribution, 174
 - Rayleigh distribution, 175
 - Weibull distribution, 174
 - discrete distributions, 173
 - table of inverses, 177
- Jacobian determinant, 147, 149
- Jacobian matrix, 148-50
- joint cumulants, 116
- joint distribution of two vectors, 196
- joint distributions, 85
 - continuous distribution, 86
 - discrete distribution, 85
 - joint probability mass function, 85
 - marginal distribution, 85
 - properties of joint distributions, 86
 - regions bounded
 - by constants, 86
 - by a function, 88
 - by functions, 88
- joint Gaussian density, 139
- joint moments of two random vectors, 202
 - cross correlation matrix, 202
 - cross covariance matrix, 202
- Kalman gain, 303
- Kalman gain vector, 303, 319
- Karhunen—Loeve transform, 381
 - continuous, 381
 - discrete, 381
 - matrix formulation, 382
- Kendall—Lee notation, 440
- Kendall rank correlation 'I, 100
 - concordant pairs, 100
 - discordant pairs, 100
 - with no ties, 100, 101
 - with ties, 101, 102, 104
- Kolmogorov axioms, 7
 - Boolean additivity, 7
 - non negativity, 7
 - normalization, 7
 - sigma additivity, 8
- Kronecker delta, 358
- kurtosis, 115
- Lagrange multipliers, 218
- Laplace transform, 361
- least mean squares algorithm (LMS), 298, 310
- least squares, 299
 - exponentially weighted, 301
- likelihood function, 213
- limit theorems, 166
 - central limit theorem, 166
 - strong law of large numbers, 166
 - weak law of large numbers, 165
- linear birth and death, 360
 - moments, 360, 364
- linear filters, 393
 - carrier frequency, 393
 - cutoff frequency, 393
 - ideal band-pass, 393
 - ideal high-pass, 393
 - ideal low-pass, 393
 - matched filters, 395
- linear regression, 214
 - coefficients, 214
 - evaluating regression coefficients, 214
 - multiple, 214
 - simple, 214, 223
 - estimated regression line, 224
 - variance of regression coefficients, 225
- linear systems-random processes, 383
 - causal system, 383
 - convolution integral, 383
 - impulse response, 383
 - review of linear systems, 383
 - state transition matrix, 384
 - transfer function, 383
- linear transformations, 206
- Little's law, 442, 448
- log likelihood function, 213
- marginal distributions, 88, 89
- Markov chains, 347, 368
 - classification of states, 352
 - absorbing state, 353
 - aperiodic, 353
 - ergodic, 353
 - intercommunicating, 352
 - irreducible, 353
 - periodic, 353
 - recurrent, 353
 - reducible, 352
 - transient, 353
 - discrete, 347
 - first passage probability, 352
 - first return probability, 353

- homogeneous, 348
- mean first passage time, 353
- mean recurrence time, 353
 - null recurrent, 353
 - positive recurrent, 353
- m-step transition probability, 348
 - steady state, 350
- properties, 356
- state occupancy probabilities, 349
- stochastic matrix, 349
- Markovian, 438
- Markov process, 342
 - AIR process, 347
 - Chapman—Kolmogorov equation, 343
 - homogeneous, 344
 - Markov property, 343
 - Poisson process, 346
 - properties, 345
 - transition probability density, 343
 - Wiener process, 346
- Markov property, 368
- Markov renewal process, 369
- martingale process, 370
 - continuous martingale, 370
 - discrete martingale, 370
 - likelihood ratio martingale, 372
 - Poisson martingale, 372
 - random walk, 371
 - Wiener martingale, 372
- mass megabucks, 29
- matched filters, 395
 - input signal to noise ratio (SNR), 396
 - nonwhite Gaussian noise, 400
 - output signal to noise ratio (SNR), 396
 - properties, 397
 - rectangular pulse, 397
 - sinusoidal pulse, 398
- matrices, 181
 - addition, 181
 - adjoint, 185
 - characteristic equation, 186
 - characteristic polynomial, 186
 - definiteness, 187
 - negative definite, 187
 - positive definite, 187
 - definition, 181
 - determinant, 182
 - properties, 183
 - diagonalization, 188
 - inverse, 184
 - properties, 186
 - multiplication, 182
 - orthogonal, 186
 - principal minors, 187
 - rank, 183
 - similarity transformation, 190, 317
 - singular, 183
 - square, 181
 - symmetric, 189
 - properties, 189
 - trace, 183
 - transpose, 182
- means and variances table, 116
- minimum mean square estimator of mean,
- minimum square error, 300, 305, 306, 309
- mixed distributions, 75
- modal matrix, 188
- modified Bessel function $J_0(z)$, 70, 141, 410
- moment generating functions, 113
 - gamma distribution, 114
 - Gaussian distribution, 114
 - negative binomial, 113
- Monty Hall problem, 14
- multiple functions of random variables, 153
- multivariable process, 312
- narrowband band-pass process, 404
 - analytic signal representation, 405
 - complex envelope, 405
 - natural envelope representation, 407
 - quadrature representation, 405
 - properties, 406
 - with additive sinusoid, 409
- negative binomial distribution, 364
- Newton's solution, 34
 - extensions, 39
 - generalizations, 37
 - Proposition 1, 33
 - Proposition 2, 34
 - Proposition 3, 34
- Newton—Raphson iteration, 444, 495
- normal equations, 214
- null space, 299, 301, 302
- operating characteristic, 237
- orthogonal increment processes, 331
- orthogonality, 300, 302, 316
- orthogonality principle, 215, 299, 310
 - review, 413
 - random processes, 413
 - random variables, 413
- Paley—Wiener criterion, 393, 416
- partial correlation coefficients, 106
 - coefficient of determination r^2 , 106, 107
- partial differentiation, 192
 - complex vectors, 193
 - real matrices, 194
 - real vectors, 190
- Pearson product moment correlation, 96
 - assumptions, 96
 - least squares, 106
- Pepys' problems, 33
 - Proposition 1, 33
 - Proposition 2, 34
 - Proposition 3, 34

- periodic random processes, 374
 - stationary, 374
- periodic stationary process, 260
- permutations, 16
 - sampling without replacement, 17
 - sampling with replacement, 17
- playing card drawing, 37
- Poisson arrivals, 437, 438, 443
- Poisson process, 321, 323, 324
 - formal derivative, 328
 - generalized, 326
 - autocorrelation, 327
 - autocovariance, 328
 - mean, 327
 - interarrival time, 321
 - distribution, 321, 340
 - variance to mean ratio, 326
 - waiting time, 323
 - distribution, 323
- Poisson white noise, 328
- Polya urn model, 347
- population, 357
- posterior error, 299, 301
- power spectral density, 287
- predictable process, 373
- pre-window, 299
- prior estimation errors, 304
- probability, 7
 - conditional, 9
 - reduced sample space, 9-11
 - density, 9
 - distribution function, 9
 - measure, 7
 - space, 7
 - total, 12
- probability mass function, 38, 51
- pseudoinverse, 211
- pure birth process, 362
 - Poisson process as a special case, 362
- pure death process, 364
 - absorbing state, 364
 - extinction probability, 366

- quadratic error surface, 193
- quadratic form, 191
- quadrature representation, 405
 - properties, 406
- queue length, 442
- queuing, 437
 - discipline, 440
 - models, 438

- random binary wave, 263
- random number generator, 169 properties, 170
- random processes, 250
 - bandpass processes, 401
 - classification of random processes, 320
 - continuous state continuous time, 320
 - continuous state discrete time, 320
 - discrete state continuous time, 320
 - discrete state discrete-time process, 320
 - definition, 250
 - distribution and density, 250
 - estimation of parameters, 273
 - continuous time processes, 273
 - autocovariance, 273
 - mean, 273
 - covariance, 276
 - variance, 273
 - discrete-time processes, 280
 - mean, 280
 - autocovariance, 280
 - estimation, 281
 - Lagrange multiplier, 280
 - higher order distributions, 254
 - independent, 256
 - means and variances, 250
 - modulation, 266
 - normalized autocovariance, 256
 - normalized cross covariance, 256
 - orthogonal, 256
 - second order moments, 255
 - autocorrelation, 255
 - autocovariance, 256
 - cross correlation, 256, 259
 - cross covariance, 256, 259
 - second order process, 255
 - normalized autocovariance, 256,
 - normalized cross-covariance, 256, 257
 - through linear systems, 385
 - cross spectral density, 386
 - output power spectral density, 386
 - threshold filter, 391
 - uncorrelated, 256
- random telegraph wave, 264
- random variable, 45
 - conditional moments, 91
 - conditional expectation, 92
 - conditional variance, 93
 - continuous, 54
 - correlation coefficient, 93
 - covariance, 93
 - definition, 45
 - degenerate, 155
 - discrete, 23
 - distribution and density, 46
 - properties, 50
 - independent, 92
 - moments, 91
 - expectation, 91
 - properties, 91
 - joint, 92
 - variance, 92
 - properties, 93

- orthogonal, 92
- uncorrelated, 92
- random vectors, 196
 - Bayes theorem, 199
 - conditional density, 205
 - independent, 203
 - inner product, 202
 - joint density, 196, 204
 - moments, 200
 - correlation matrix, 201
 - properties, 203
 - covariance matrix, 202
 - properties, 203
 - expectation vector, 200
 - orthogonal, 203
 - outer product, 203
 - uncorrelated, 203
- random walk process, 320, 333
 - elementary, 331
 - symmetric, 332
 - two absorbing barriers, 333
 - unrestricted, 333
- Rayleigh density, 140, 410
- Rayleigh distribution, 140, 150, 409
- recursive estimation, 426
- recursive least squares algorithm (RLS), 302
- recursive update, 303, 304
- regression coefficients, 214
- relaxation parameter, 311, 313, 314, 318
- renewal equation, 368, 369
- renewal function, 367, 368
- renewal process, 366
 - Poisson process, 366
- Rice's density, 141, 410
- Rice's distribution, 141
- roots of $D(z) = 0$, 444
- Rouches theorem, 444

- sample space, 7
 - continuous, 7
 - discrete, 7
- sample variance, 220
- scatter diagram, 104
- semi-Markov, 368, 369
- semi-Markov kernel, 369
- set, 1
 - cardinality, 2
 - unions and intersections, 3
 - Cartesian product, 4
 - complement, 4
 - De Morgan's laws, 5
 - difference, 4
 - element, 1
 - empty set, 1
 - equal, 2
 - event, 6
 - finite, 1
 - inclusion—exclusion principle, 3
 - intersection, 2
 - mutually exclusive, 3
 - null, 8
 - partition, 5
 - power set, 2
 - relative complement, 4
 - subset, 2
 - symmetric difference, 4
 - table of set properties, 5
 - union, 2
 - universal, 1
 - Venn diagram, 1
 - shaping filter, 386
 - signal to noise ratio, 162, 395
 - significance level, 218, 226, 231
 - similarity transformations, 190, 317
 - skewness, 115
 - sojourn time, 368
 - solution of linear equations, 185
 - Spearman rank correlation, 97, 99
 - alternate derivation, 98
 - assumptions, 97
 - with no ties, 97
 - with ties, 100
 - special methods, 175
 - Box—Mueller transformation, 176
 - Gaussian distribution, 176
 - Rayleigh distribution with phase, 177
 - spectral density continuous time, 287
 - bandlimited process, 290
 - bandpass process, 291
 - cross spectral density, 288
 - power spectral density, 288
 - alternate form, 293
 - estimation, 293
 - periodogram, 293
 - properties, 292
 - white noise, 291
 - Wiener—Khinchine theorem, 288
 - spectral density discrete time, 294
 - alternate form, 296
 - discrete white noise, 297
 - power spectral density, 294
 - spectral factorization, 417
 - additive factoring, 418
 - multiplicative factoring, 418
 - spectral norm, 309
 - standard error, 220
 - state diagram, 358, 362, 364
 - birth and death process, 358
 - birth process, 362
 - death process, 364
 - state transition, 348
 - stationary probability, 360
 - stationary random processes, 258
 - autocovariance, 259
 - autocorrelation, 259
 - cross correlation, 259

- cross covariance, 259
 - first order, 259
 - independent, 259
 - mean, 259
 - normalized autocovariance, 259
 - normalized cross covariance, 259
 - n th order stationary, 258
 - orthogonal, 259
 - periodic, 260
 - properties of correlation functions, 259
 - second order, 258
 - strict sense, 258
 - uncorrelated, 259
 - variance, 259
 - wide sense, 258
 - steepest descent, 310, 311
 - stochastic estimation algorithm, 455
 - expectation step, 455
 - maximization step, 456
 - strong law of large numbers, 166
 - submartingale, 370
 - decomposition, 374
 - subscriber load, 438
 - supermartingale, 370
 - suppressor variable, 107
 - system state, 438, 439, 442
- tables, 463
- computation of roots of $D(z) = 0$, 494
 - cumulative binomial, 488
 - cumulative Gaussian, 467
 - cumulative Poisson, 484
 - Fourier transform, 463
 - inverse chi-square, 474
 - inverse Gaussian, 472
 - inverse student-t, 481
 - tail of the distribution, 445
 - tails probability, 155
 - tapped delay line, 298
 - formulation, 298
 - transversal filter, 298
 - tap weight coefficients, 304
 - telephone traffic, 438
 - erlang, 438
 - intensity, 438
 - load, 438
 - teletraffic models, 437
 - exponentially distributed, 437
 - time-averaged probabilities, 438
 - toss of 6n dice, 36
 - total input energy, 318
 - total probability for densities, 83
 - transition probability, 358
 - transition rates, 358
 - transmission tomography, 452
 - computer simulation, 458
 - pathlength matrix, 459
 - backward, 459
 - forward, 459
 - phantom description, 460
 - probability model, 453
 - reconstructed images, 458
 - cross sectional histograms, 460
 - RMS error, 460
 - truncated Poisson distribution, 439
 - trunks, 437
 - two functions of two random variables, 143
 - auxiliary random variable, 150
 - conditions, 143
 - derivation of student-t, 153
 - type I error, 231
 - false alarm, 231
 - producer's risk, 231
 - type II error, 231
 - consumer's risk, 231
 - missed alarm, 231
- unbiased of estimator, 304
- uncorrelated increment process, 331
- uniform distribution, 169
- utilization factor, 449
- variance of the estimated regression line, 223
- variance to mean ratio, 326
- vector Gaussian random variables, 204
- waiting room, 440
- waiting time, 366, 441, 446
 - complementary, 446
 - customers, 446
 - distribution, 446
 - expected, 442, 448
- weak law of large numbers, 165
- weighted least squares, 301
- weighted Poisson process, 327
- weight error matrix, 314
- weighting coefficients determination, 299
- white Gaussian noise, 339
- white measurement noise, 304
- whitening filter, 417
- white noise process, 259
- Wiener filtering, 414
 - causal, 416
 - filtering, 414, 420
 - noncausal, 414
 - prediction, 414, 422
- Wiener—Hopf integral equation, 417
 - filtering solution, 418
 - prediction-smoothing solution, 420
- Wiener process, 340
 - conditions, 340
 - derivation, 341
 - formal derivative, 342
 - properties, 341
 - smoothing, 414, 422
- Wiener solution, 299, 305, 311

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Venkatarama Krishnan, PhD, is Professor Emeritus in the Department of Electrical and Computer Engineering at the University of Massachusetts at Lowell having previously taught at Smith College, Indian Institute of Science, Polytechnic University in Brooklyn, University of Pennsylvania, Villanova University and Princeton University. He has served as a consultant to the Dynamics Research Corporation, the U.S. Department of Transportation, and Bell Laboratories. Dr. Krishnan's research includes estimation of steady-state queue distribution, tomographic imaging, aerospace, control, communications, and stochastic systems. Dr. Krishnan is a senior member of the IEEE and listed in Who is Who in America.

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