Mathematics of Melakartha Ragas in Carnatic Music

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Introduction

Indian music was developed and practiced nearly three thousand years ago. It consisted of musical styles and schools evolved by blending Vedic chants and folk music. The South Indian Classical music practiced in Tamil Nadu, Andhra Pradesh, Kerala and Karnataka is called Carnatic Music. It starts from the diatonic scale of seven note, octave-repeating scale comprising of the swaras Sa, Ri, Ga, Ma, Pa, Da, Ni along with five semi tones. It was systematized in the fifteenth century and considerably embellished in the seventeenth century by Venkatamakhin or Venkateswara Dikshitar from Karnataka in his landmark work 'Chaturdandi Prakasika' (1650) [1] and continues to be refined. The compositions are called ragas [4], which may consist of either all the seven swaras or more or less. It is the fertile imagination of the South Indian musicologists that has evolved into the nomenclature of the ragas particularly the ones consisting of all the seven swaras and the upper octave Sa, which are called Melakartha ragas with eight swaras in the scale.

Ragas

A raga [4] consists of a minimum of five or more musical swaras upon which a melody is constructed. The swaras are approached and rendered in musical phrases so that they convey moods for a particular time or season of the year. Each raga will have a set of swaras for ascending called *arohana* and a set for descending called *avarohana*. The Indian classical music is always set in a particular raga. There are two distinct classes of Carnatic music ragas. They are the 72 Janaka ragas meaning parent ragas consisting of all the seven swaras along with the first swara of the octave, and a number of Janya ragas, which are derived form the Janaka ragas by omitting some swaras or by adding extra swaras.

Swaras in Carnatic Music

The seven swaras [4] in an octave Sa, Ri, Ga, Ma, Pa, Da, Ni are in strict ascending order of frequencies and will be written as S, R, G, M, P, D, N with the first S and the fifth P are fixed and the other five, R, G, M, D, N are separated by a half notes with names as shown in Table -1:

	Swaras			Names of Swaras	Equal	Combination
1	Sa	1	S	Shadjamam - Lower		
2	Ri	2	R1	Shuddha Rishabham		G1, G2, G3
		3	R2	Chatusruti Rishabham	= G1	G2, G3
		4	R3	Shatsruti Rishabham	= G2	G3
3	Ga	5	G1	Shuddha Gandharam		R1
		6	G2	Sadharana Gandharam		R1, R2
		7	G3	Antara Gandharam		R1, R2, R3
4	Ma	8	M1	Shuddha Madhyamam		
		9	M2	Prati Madhyamam		
5	Pa	10	P	Panchamam		
6	Da	11	D1	Shuddha Deivatam		N1, N2, N3
		12	D2	Chatusruti Deivatam	= N1	N2, N3
		13	D3	Shatsruti Deivatam	= N2	N3
7	Ni	14	N1	Shuddha Nishadam		D1
		15	N2	Kaisika Nishadam		D1, D2
		16	N3	Kakali Nishadam		D1, D2, D3
8	Sa	17	S	Shadjamam - Upper		

Table -1

Among these half notes, R2 = G1, R3 = G2, D2 = N1, D3 = N2 as shown in Table-1. Essentially there are 7 main swaras S, R1, G1, M1, P, D1, N1 and 5 half swaras, R2, G2, M2, D2, N2. The swaras S, R, G, M are called *Purvanga* and the swaras P, D, N are called *Uttaranga*.

Melakartha Ragas

For a raga to be considered a Melakartha Raga [5] the following conditions must be satisfied.

1. It must have all the seven swaras of the octave.
2. The upper Shadjamam S of the succeeding octave must be included making it an eight swara sequence, S, R, G, M, P, D, N, S. Ragas having only seven swaras without the upper Shadjamam will not be considered a Melakartha Raga.
3. The same swaras must be in the arohanam and avarohanam.
4. All swaras must be strictly increasing in frequency for arohanam and strictly decreasing for avarohanam.
5. If all the above four conditions are satisfied then the raga is called a Krama Sampoorna Raga.

In constructing a scale for a Melakartha Raga, the first, fifth and the eighth spaces are occupied by the fixed swaras, S-lower, P and S-upper. The other five spaces are obtained by permutations of the remaining fourteen swaras following the strict conditions enunciated above. To start the permutation process, the Shuddha Madhyamam M1 occupies the fourth place. If the second place is occupied by the Shuddha Rishabham R1, then there are 3 ways in which the three Gandharams G1, G2, G3 can occupy the third place. If Chatusruthi Rishabham R2 occupies the second space, then by condition 4 there can be only 2 ways in which the third space can be occupied by the two Gandharams G2, G3. Finally, if the second space is occupied by the Shatsruti Rishabham R3, then by condition 4 only G3 can occupy the third space. Since all these occupancies are independent there are 3 x 2 x 1 = 6 ways in which this arrangement can be accomplished. In a similar manner, if the sixth space is occupied by any of the three Deivatams, then the seventh and final space can be occupied by the Nishadams in 3 x 2 x 1 = 6 ways. Again, since these 2 sets of placements are independent there are 6 x 6 = 36 ways for the Shuddha Madyaman M1. For the Prati Madyamam M2 in the fourth place there will be another 36 ways giving a total of 72 different combinations of scales. Thus, there are only 72 Melakartha ragas that are possible.

Rank Ordering of the Melakartha Ragas

The next step was to rank order the scales of these ragas in such a manner that an algorithm can be constructed so that the scale can be obtained directly from the rank order. The first level of 1-36 ragas was based on the scales containing the Shuddha Madhyama M1 and the next level of 37-72 ragas was based on the scales containing the Prati Madhyama M2. Thus if the raga has a rank order of 46 it is immediately known that it contains the swara M2. The second level of ranking was to arrange the scales into 12 groups of 6 scales called Chakras having Purvanga swaras S, R, G, and M within each Chakra having the same combinations of R and G as shown in Table-2. The changes in the Uttaranga swaras D and N in each Chakra will be the same with P remaining fixed. This type of ranking makes it amenable for constructing a simple algorithm to determine the scale for each raga from the rank order.

The Carnatic musicologists have named these Chakras in such a way that the number of the Chakra can be obtained from the name [6]. The Chakras corresponding to M1 are : 1. Indu (one moon), 2. Netra (two eyes), 3. Agni (three fires Ahvania, Dakshina and Garhapadya), 4. Veda (four Vedas, Rig, Yajur, Sama and Atharvana), 5. Bana (five arrows Lotus, Mango, Jasmine, Ashoka and Lily), 6. Rutu, (six seasons Vasantha, Greeshma, Varsha, Sharad, Sisira and Hemantha). Chakras corresponding to M2 are 7. Rishi (seven rishis Goutama, Vishwamitra, Kashyapa, Jamadagni, Bharadwaja, Atri and Vasishta), 8. Vasu (eight Vasus Aapa, Dhruva, Soma, Dava, Pratyusha, Anila, Anala and Prabhasa), 9. Brahma (nine

RANK ORDERING OF SCALES														
SHUDDHA MADHYAMA – M1								PRATHI MADHYAMA – M2						
1. INDU CHAKRA								7. RISHI CHAKRA						
1	S	R1, G1	M1	P	D1	N1		37	S	R1, G1	M2	P	D1	N1
2						N2		38						N2
3						N3		39						N3
4					D2	N2		40					D2	N2
5						N3		41						N3
6					D3	N3		42					D3	N3
2. NETRA CHAKRA								8. VASU CHAKRA						
7	S	R1, G2	M1	P	D1	N1		43	S	R1, G2	M2	P	D1	N1
8						N2		44						N2
9						N3		45						N3
10					D2	N2		46					D2	N2
11						N3		47						N3
12					D3	N3		48					D3	N3
3. AGNI CHAKRA								9. BRAHMA CHAKRA						
13	S	R1, G3	M1	P	D1	N1		49	S	R1, G3	M2	P	D1	N1
14						N2		50						N2
15						N3		51						N3
16					D2	N2		52					D2	N2
17						N3		53						N3
18					D3	N3		54					D3	N3
4. VEDA CHAKRA								10. DISI CHAKRA						
19	S	R2, G2	M1	P	D1	N1		55	S	R2, G2	M2	P	D1	N1
20						N2		56						N2
21						N3		57						N3
22					D2	N2		58					D2	N2
23						N3		59						N3
24					D3	N3		60					D3	N3
5. BANA CHAKRA								11. RUDRA CHAKRA						
25	S	R2, G3	M1	P	D1	N1		61	S	R2, G3	M2	P	D1	N1
26						N2		62						N2
27						N3		63						N3
28					D2	N2		64					D2	N2
29						N3		65						N3
30					D3	N3		66					D3	N3
6. RUTU CHAKRA								12. ADITYA CHAKRA						
31	S	R3, G3	M1	P	D1	N1		67	S	R3, G3	M2	P	D1	N1
32						N2		68						N2
33						N3		69						N3
34					D2	N2		70					D2	N2
35						N3		71						N3
36					D3	N3		72					D3	N3
	In each Chakra Purvanga swaras S, R, G, M are the same and the changes in Uttaranga swaras P, D and N are only in D and N													

Table-2

Brahmas (Atri, Angirasa, Brighu, Daksha, Kashyapa, Pulaha, Marichi, Vasishta and Pulastya). 10. Disi (ten masters of directions (Ashtadigpalas) Indra, Agni, Yama, Niruddhi, Varuna, Vayu, Kubera, Isana, Akasha and Patala), 11. Rudra (eleven Sivas Aja, Dwasha, Ekapada, Triambaka, Aparajitha, Isana, Tribhuvana, Shambhu, Hara, Rudra, Ahirputnia), 12. Aditya (twelve Suryas Poosha, Bhaskara, Marichi, Arka, Khaga, Surya, Mitra, Aditya, Ravi, Bhanu, Savitha and Hiranyagarbha).

Table-2 shows the ranking of the scales 1-36 and the Chakras 1-6 for Shuddha Madyama M1, and scales 37-72 and Chakras 7-12 for Prati Madhyama M2.

Determination of the Scale from Rank Order

Having properly ranked the scales, an algorithm could be established for determining them from the rank order. Since the ranking does not start with zero, 1 is subtracted from the rank numbers 1-36 for M1 and then divided by 6 corresponding to the number of scales in each Chakra. The resulting quotient will yield the number of the Chakra, which having the same R, G combinations, will give the Purvanga swaras of the scale. The remainder from the division will give the D, N combinations, which are the Uttaranga swaras of the scale. These combinations are shown in Table-3 for the quotient and the remainder. Similarly 36 is first subtracted from the rank numbers 37-72 for M2 and applying the same procedure yields the corresponding scales for M2. The flow chart is shown in Table-4.

Quotient	Purvanga Swara Combination	Remainder	Uttarnaga Swara Combination
0	R1, G1	0	D1, N1
1	R1, G2	1	D1, N2
2	R1, G3	2	D1, N3
3	R2, G2	3	D2, N2
4	R2, G3	4	D2, N3
5	R3, G3	5	D3, N3

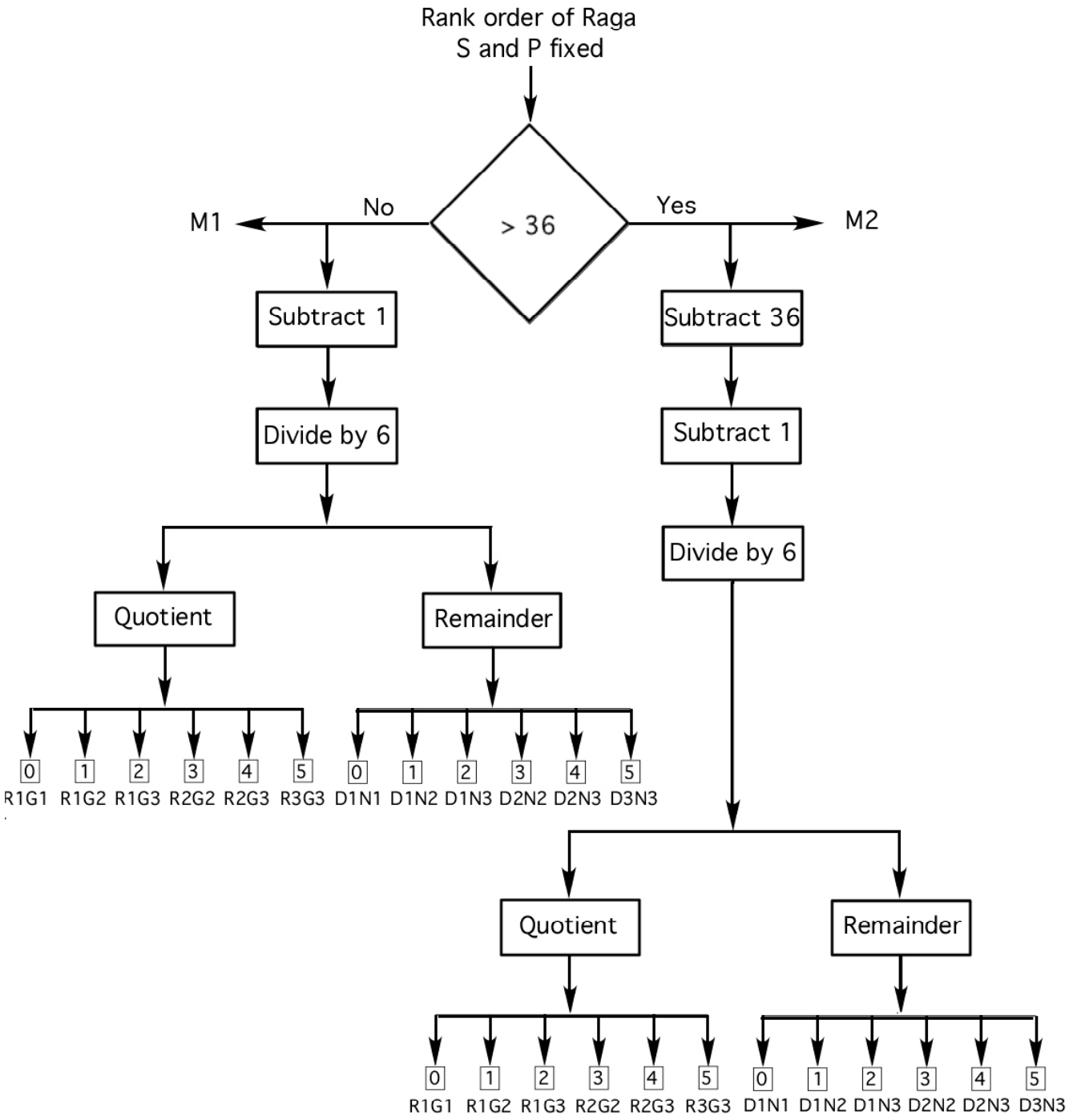
Table-3

As an example if the rank order is 28 then being less than 37 the scale is in Shuddha Madyama M1 and 28 – 1 = 27 divided by 6 yields the quotient 4 and remainder 3. The Purvanga swaras from Table-3 are R2, G3 and the Uttaranga swaras are D2, N2. Thus the complete scale is S, R2, G3, M1, P, D2, N2, S for rank order 28. Similarly, if the rank order is 56 then being greater than 36 the scale is in Prati Madyama M2, and 56 – 36 – 1 = 19 divided by 6 yields the quotient 3 and remainder 1. The complete scale from Table-3 is S, R2, G2, M2, P, D1, N2, S for rank order 56.

For rank order 1 the Quotient Remainder result is 1 – 1 = 0/6 = 0, 0 and the scale is S, R1, G1, M1, P, D1, N1, S. For rank order 72 the Quotient Remainder result is 72 – 36 – 1 = 35/6 = 5, 5 and the scale is S, R3, G3, M2, P, D3, N3, S.

Even though the Carnatic musicologist Venkatamakhin postulated in 1650 the maximum possibility of 72 Melakartha ragas, there were only 19 ragas in vogue at that time. Many composer musicians like Muthswami Dikshitar (1775-1835) thought that this classification was too restrictive and used their own classifications. Nevertheless, many composer musicians adopted this scheme and composed krithis in all 72 ragas. One of the earliest musicians who composed in all 72 ragas was the doyen of Tamil Compositions, Koteeswara Iyer (1870-1938). He lived in Madras for two years and presented my mother with his second volume of Melakartha ragas before he passed away in 1938 at the age of about 68.

The next problem was to name these ragas so that the rank ordering can be obtained from the name. The encryption method used to name the ragas was the Katapayadi scheme, which the ancient Hindus used to encrypt numbers. This scheme is discussed in the next section.



Flow Chart for determining Scale from Rank Order of Melakartha Ragas – Table-4

Katapayadi Scheme

The number system as enunciated by the ancient Hindus used the place values from left to right as given by अङ्कानाम् वामतो गतिः [2] meaning numbers go from left to right. In the present day the place values run from right to left. Thus the number 365 in the Katapayadi scheme [3] would be written as 563. If this

scheme is used to decrypt then the result must be inverted to convert to the present scheme. This scheme based on the Sanskrit alphabets has the following conditions.

1. Only consonants have numerals assigned to them as shown in Table 5.
2. All unattached vowels like ah (अ) and uh (उ) are assigned zero.
3. In case of compound alphabets, only consonants attached to a vowel will have a value. For example, in kshvaa (क्ष्वा) formed by ksh (क्ष) + va (व) + a (अ), the only consonant standing with a vowel is va (व) and is given a value 4 from Table-5 whereas ksh (क्ष) an unattached consonant and ah (अ) a stand-alone vowel do not enter into consideration.
4. There is no encryption for decimals.

	1	2	3	4	5	6	7	8	9	0
Ka	क	ख	ग	घ	ङ	च	छ	ज	झ	ञ
Ta	ट	ठ	ड	ढ	ण	त	थ	द	ध	न
Pa	प	फ	ब	भ	म					
Ya	य	र	ल	व	श	ष	स	ह		

Table 5

An interesting example is the value of Pi (π) encrypted in the Katapayadi scheme in a book Sadratnamala (सद्रत्नमाला) (1819) on astronomy by Raja Sankara Varma of Tellichery (1774-1839) [3]: भद्राम् बुद्धि सिद्ध जन्म गणित श्रद्धा स्म यद् भूपगी: and is decrypted in Table -6.

भ	द्	रा	म्	बु	द्	धि	सि	द्	ध	ज	न्	म	ग	णि	त	श्	र	द्	धा	स्	म	य	द्	भू	प	गी
4	-	2	-	3	-	9	7	-	9	8	-	5	3	5	6	-	2	-	9	-	5	1	-	4	1	3

Table – 6

The value of Pi is 3 1415926535 8979324 correct upto 16 places of decimal rounded off at the 17th place, the correct value upto 20 places of decimal being 3.1415926535 8979323846. The decimal point does not come into play.

Nomenclature of Melakartha Ragas

Having established mathematically that only 72 Melakartha Ragas are possible, the musicologists had to devise a convenient method of naming them so that one can determine from the name the rank in the Melakartha hierarchy and from the rank the scale as explained in the previous section. They exercised considerable ingenuity in creating names using the Katapayadi scheme. For example, the Melakartha Raga Shanmukhapriya (षण्मुखप्रिया) has the rank 56. Referring to Table-5 the letter ष is assigned 6 and the letter म is assigned 5 with the letter ण् omitted being an unaccompanied consonant, and by inverting the order the rank 56 is obtained. Similarly the raga Vachaspathi (वाचस्पति) has the rank 64 and referring to Table-5 the letter व is 4 and the letter च is 6 with the stand-alone vowel अ omitted. Inverting these letters the rank 64 is obtained. The complete list of 72 ragas grouped under Chakras is given in English and Sanskrit in Table-9 with the scales and the rank. However, the rank ordering of 16, 48, 54 and 66 of the ragas Chakravaakam (चक्रवाकम्), Divyamani (दिव्यमणि), Vishwaambari (विश्वाम्बरि) and Chitraambari (चित्राम्बरि) is based on their older names Thoyavegavaahini (तोयवेगवाहिनि 16), Jeevanthini (जीवन्तिनि 48), Vamsaavathi (वंशावति 54) and Chaturangini (चतुरङ्गिनि 66) as shown in Table-9.

MELAKARTHA RAGAS [Shadjam (S) and Panchamam (P) fixed] - I																
#	SHUDDHA MADHYAMA – M1						RAGA NAMES		RAGA NAMES	PRATHI MADHYAMA – M2						#
1. INDU CHAKRA									7. RISHI CHAKRA							
1	S	R1, G1	M1	P	D1	N1	Kanakaangi कनकाङ्गि		Saalagam सालगम्	N1	D1	P	M2	G1, R1	S	37
2						N2	Ratnaangi रत्नाङ्गि		Jalaarnavam जलार्णवम्	N2						38
3						N3	Gaanamurthy गानमूर्ति		Jhaalavaraali झालवरालि	N3						39
4					D2	N2	Vanaspathi वनस्पति		Navaneetham नवनीतम्	N2	D2					40
5						N3	Maanavathi मानवति		Paavani पावनि	N3						41
6					D3	N3	Thaanaroopi तानरूपि		Raghupriya रघुप्रिया	N3	D3					42
2. NETRA CHAKRA									8. VASU CHAKRA							
7	S	R1, G2	M1	P	D1	N1	Senaavathi सेनावति		Gavaambodi गवाम्बोधि	N1	D1	P	M2	G2, R1	S	43
8						N2	Hanumatodi हनुमत्तोडि		Bhaavapriya भावप्रिया	N2						44
9						N3	Dhenuka धेनुका		Shubhapantuvaraali शुभपन्तुवरालि	N3						45
10					D2	N2	Naatakapriya नाटकप्रिया		Shadvidamaargini षड्विदमार्गिणि	N2	D2					46
11						N3	Kokilapriya कोकिलप्रिया		Suvarnaangi सुवर्णाङ्गि	N3						47
12					D3	N3	Roopavathi रूपवति		Divyamani दिव्यमणि = जीवन्तिनि	N3	D3					48
3. AGNI CHAKRA									9. BRAHMA CHAKRA							
13	S	R1, G3	M1	P	D1	N1	Gaayakapriya गायकप्रिया		Dhavalaambari धवलाम्बरि	N1	D1	P	M2	G3, R1	S	49
14						N2	Vakulaabharanam वकुलाभरणम्		Naamanaarayani नामनारायणि	N2						50
15						N3	Mayaamaalavgowlam मायामालवगौलम्		Kaamavardhini कामवर्दनि	N3						51
16					D2	N2	Chakravaakam छक्रवाकम् = तोयवेगवाहिनि		Raamapriya रामप्रिया	N2	D2					52
17						N3	Suryakaantham सूर्यकान्तम्		Gamanaashrama गमनाश्रम	N3						53
18					D3	N3	Haatakaambari हाटकाम्बरि		Vishwaambari विश्वाम्बरि = वंशावति	N3	D3					54

Table - 9

MELAKARTHA RAGAS [Shadjam (S) and Panchamam (P) fixed] - II																
#	SHUDDHA MADHYAMA – M1						RAGA NAMES		RAGA NAMES	PRATHI MADHYAMA – M2						#
4. VEDA CHAKRA									10. DISI CHAKRA							
19	S	R2, G2	M1	P	D1	N1	Jankaaradhwani झंकारध्वनि		Shyamalaangi शामलाङ्गि	N1	D1	P	M2	G2, R2	S	55
20						N2	NataBhairavi नटभैरवि		Shanmukhapriya षण्मुखप्रिया	N2						56
21						N3	Keeravaani कीरवाणि		Simhendramadhyamam सिंहेन्द्रमध्यमम्	N3						57
22					D2	N2	Karaharapriya खरहरप्रिया		Hemavathi हेमावति	N2	D2					58
23						N3	Gowrimanohari गौरीमनोहरी		Dharmavathi धर्मवति	N3						59
24					D3	N3	Varunapriya वरुणप्रिया		Neethimathi नीतिमति	N3	D3					60
5. BANA CHAKRA									11. RUDRA CHAKRA							
25	S	R2, G3	M1	P	D1	N1	Maararanjani माररंजनि		Kaanthamani कान्तामणि	N1	D1	P	M2	G3, R2	S	61
26						N2	Charukesi चारुकेशि		Rishabhapriya रिषभप्रिया	N2						62
27						N3	Sarasaangi सरसाङ्गि		Lathaangi लताङ्गि	N3						63
28					D2	N2	Harikambodhi हरिकाम्बोदि		Vaachaspathi वाचस्पति	N2	D2					64
29						N3	Dheerasankarabharanam धीरशङ्कराभरणम्		MechaKalyaani मेचकल्याणि	N3						65
30					D3	N3	Nagaanandini नागानंदिनि		Chitraambari चित्राम्बरि = चतुरङ्गिनि	N3	D3					66
6. RUTU CHAKRA									12. ADITYA CHAKRA							
31	S	R3, G3	M1	P	D1	N1	Yaagapriya यागप्रिया		Sucharitra सुचरित्रा	N1	D1	P	M2	G3, R3	S	67
32						N2	Raagavardhani रागवर्धनि		Jothi Swaroopini जोतिस्वरूपिणि	N2						68
33						N3	Gaangeyabhushani गांगेयभूषणि		Dhathuvardani धातुवर्धनि	N3						69
34					D2	N2	Vagadheeswari वागदीश्वरि		Naasikaabhushani नासिकाभूषणि	N2	D2					70
35						N3	Shoolini शूलिनि		Kosalam कोसलम्	N3						71
36					D3	N3	Chalanaata चलनाट		Rasikapriya रसिकप्रिया	N3	D3					72

Table – 9 Continued

Conclusion

It is remarkable that the South Indian Musicologists have developed carnatic music to a very high degree of sophistication with 72 major ragas and hundreds of derived ragas called Janya ragas. Some of the Janya ragas like Bhairavi is more in use than the Janaka raga NataBhairavi. The globally famous Janya raga Mohanam is as dominant as the Janaka raga Harikambodhi and uses the pentatonic scale, S R2 G3 P D2 S for arohanam and avarohnam, that is very prevalent in the Far East countries like Japan and China.

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Author Biography

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