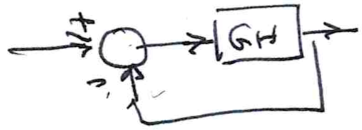
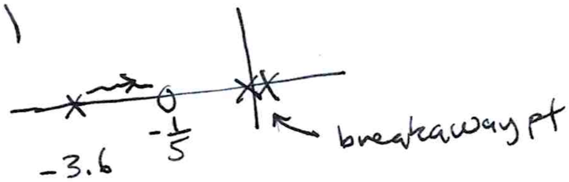
1 (a) $\dfrac{Y}{X} = \dfrac{GH}{1+GH}$



(b)



(c) $\sigma = \dfrac{+0+0-3.6+1/5}{3-1} = \dfrac{-3.4}{2} = -1.7$

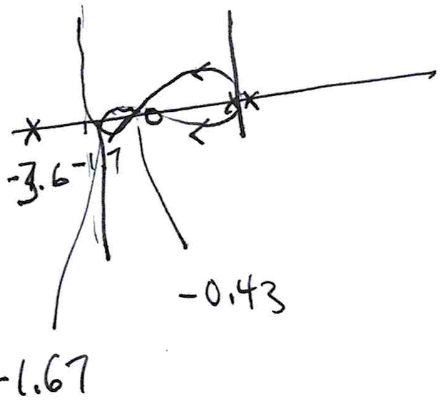
$\measuredangle = \pm\dfrac{\pi}{2}$

(d) $s^3 + 3.6s^2 + ks + k/5$

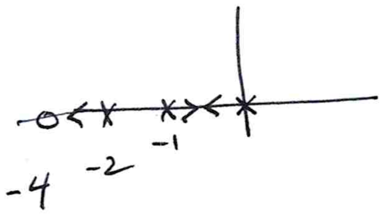
(e) $k = \dfrac{-5s^3 + 18s^2}{5s+1}$ $\dfrac{dk}{ds} = 0 = 50s^3 + 105s^2 + 36s = 0$

$s = \begin{cases} -1.67 \\ -.43 \\ 0 \end{cases}$

(f)



(2) (a) $GH = \dfrac{k(s+4)}{s(s+2)(s+1)}$



(b) $\phi = \dfrac{\pm(2n+1)}{3-1} = \pm\dfrac{\pi}{2}$

$\sigma = \dfrac{0-2-1-(-4)}{3-1} = \dfrac{1}{2}$

(c) $GH + 1 \Rightarrow s(s+2)(s+1) + k(s+4) = 0$

$s^3 + 3s^2 + (k+2)s + 4k = 0$

s^3	1	$k+2$
s^2	3	$4k$
s^1	$\dfrac{3(k+2)-4k}{3}$	
s^0	$4k$	

$\Rightarrow R(s) = 3s^2 + 4k = 0 \Rightarrow s^2 = -\dfrac{8}{3}$

$\Longrightarrow \quad -k + 2 = 0 \Rightarrow \boxed{k = 2}$

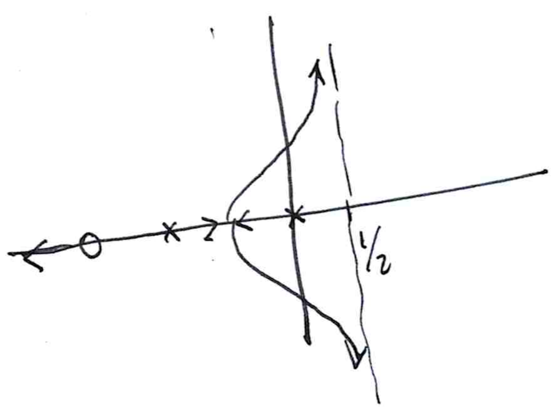
$\boxed{s = \pm i\sqrt{\dfrac{8}{3}}}$

(d) $\dfrac{dk}{ds} = 0 = 2s^3 + 15s^2 + 24s + 8 = s^3 + \frac{15}{2}s^2 + 12s + 8/2$

$\boxed{s = -0.45}$

$s = \begin{cases} -1.6 \\ -5.4 \\ \cancel{-2.2}\ -0.45 \end{cases}$

(f)



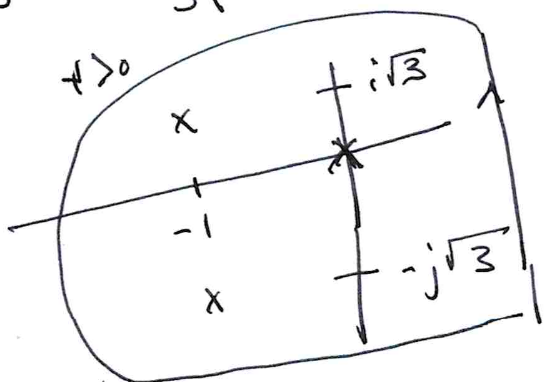
3.

$$\frac{Y}{U} = H(s) = \frac{4}{s^2+2s+4} \qquad U \to \boxed{} \to Y$$

(a) $U(s) = \frac{1}{s}$

$$Y = \frac{4}{s(s^2+2s+4)} = \cancel{\frac{4}{s[+\sqrt{3}i+]}}$$

$$y(s) = \frac{4}{s(s+1+i\sqrt{3})(s+1-i\sqrt{3})}$$



$t<0$ $y(t)=0$

$t>0$

$$y(t) = sY(s)e^{st}\Big|_{s=0} + Y(s)e^{st}(s+1+i\sqrt{3})\Big|_{s=-1-i\sqrt{3}}$$

$$+ Ye^{st}(s+1-i\sqrt{3})\Big|_{s=-1+i\sqrt{3}}$$

$$y(t) = 1 - e^{-t}\left[\cos(\sqrt{3}t) + \frac{\sin(\sqrt{3}t)}{\sqrt{3}}\right]$$

(b)

$$\dot{\underline{x}} = [A]\underline{x} + [B]\underline{u} \qquad \frac{Y(s)}{U(s)} = \frac{4}{s^2+2s+4}$$

$$\underline{y} = [C]\underline{x}$$

$$\frac{Y}{X}\,\frac{X}{U} \Rightarrow \frac{Y}{X} = 1 \Rightarrow C = [1, 0]$$

$$\frac{X}{U} = \frac{4}{s^2+2s+4} \Rightarrow A = \begin{bmatrix} 0 & 1 \\ -4 & -2 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$$

eigenvalues and vectors

$$\lambda_1 = -\sqrt{3}\,i - 1 \qquad \lambda_2 = -1 + i\sqrt{3}$$

$$\underline{e}_1 = \begin{pmatrix} 1 \\ -1-i\sqrt{3} \end{pmatrix} \qquad \underline{e}_2 = \begin{pmatrix} 1 \\ -1+i\sqrt{3} \end{pmatrix}$$