NAME______________________________

University of Massachusetts Lowell
Department of Electrical and Computer Engineering

1. Consider the uncontrolled system model as $\dot{x} = u$ The performance is measured over time interval (0,1) as

$$PI = \int_0^1 \frac{1}{2}\left[x^2 + u^2\right]dt$$

a. Determine the state-function of Pontryagin H.

b. Determine the optimal input u^o.

c. Determine the H^o.

d. Determine the equations governing the controlled system in terms of x and λ.

e. Find the optimal input given $x(0) = 0$ and $x(1) = 1/2$.

2. Consider the system that minimizes PI

$$PI = \int_0^1 \frac{u^2}{2}\, dt$$

subject to the constraints

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -2x_1 - 3x_2 + u$$

using the State Function of Pontryagin.

a. Determine the state function of Pontryagin H.

b. Find the optimal input u^0 and H^0

c. Find the matrix A that will yield the governing equations

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{\lambda}_1 \\ \dot{\lambda}_2 \end{bmatrix} = [A]\begin{bmatrix} x_1 \\ x_2 \\ \lambda_1 \\ \lambda_2 \end{bmatrix}$$

3. Consider the linear system governed by the state equations:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -7 & -8 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ K \end{bmatrix} r(t) \qquad y = [1\ Q]\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

a. Find the eigenvalues and eigenvectors.

b. Find the state transition matrix.

c. Given that the Laplace transform of $r(t)$ is equal to $R(s)$ find the transfer function $Y(s)/R(s)$. Initial conditions equal zero.