University of Massachusetts Lowell
Department of Electrical and Computer Engineering
EECE 4130 Linear Feedback

Problem set 4

1. Consider the system

$$\dot{\underline{x}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 2 & 1 & -2 \end{bmatrix} \underline{x} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u(t)$$

$$y = [1\ 0\ 1]\underline{x}$$

a. Determine the eigenvalues and eigenvectors of the system

b. Determine the state-transition matrix

c. Determine the $\underline{x}(t)$ in terms of $\underline{x}(0)$ and $u(t) = 0$.

d. Is the system controllable.

e. Is the system observable.

2. Consider the system

$$\dot{\underline{x}} = \begin{bmatrix} -1 & 0 & 0 \\ -2 & -2 & 0 \\ 1 & 1 & -3 \end{bmatrix} \underline{x} + \begin{bmatrix} -1 & b \\ 1 & 0 \\ 1 & 1 \end{bmatrix} \underline{u}(t)$$

$$\underline{y} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & f & g \end{bmatrix} \underline{x}$$

1. Determine the eigenvalues and eigenvectors of the system
2. Determine the state-transition matrix
3. Determine the conditions for controllability
4. Determine the conditions for observability

3. Determine the root loci for the closed-loop unity negative feedback system $H(s) = 1$

$$G(s) = \frac{K}{s(s+1)(s^2+4s+5)}$$

a. Evaluate the real-line root locus and directions of closed-loop pole migration with increasing gain K.

b. Determine the asymptotes and intercept

c. Determine the conditions for system stability, $j\omega$ axis crossing and gain.

d. Determine breakin/breakaway points

e. Determine arrival/departure angles

4. Determine the root loci for the closed-loop unity negative feedback system $H(s) = 1$

$$G(s) = \frac{K(s+9)}{s(s^2+4s+11)}$$

a. Evaluate the real-line root locus and directions of closed-loop pole migration with increasing gain K.

b. Determine the asymptotes and intercept

c. Determine the conditions for system stability, $j\omega$ axis crossing and gain.

d. Determine breakin/breakaway points

e. Determine arrival/departure angles

f. By adjustment of the gain K locate the closed-loop poles on the root loci such that the dominant closed-loop poles have a damping factor equal to 0.5.