4B

Inhomogeneous Solution

$$\dot{\underline{x}} = A\underline{x} + B\underline{u}$$

$$\boxed{\underline{x}(t) = \Phi\,\underline{x}(0) + \int_{-\infty}^{t} \Phi(\tau)\,B\,\underline{u}(t-\tau)\,d\tau}$$

proof!

$$\dot{\underline{x}} = A\underline{x} + B\underline{u}$$

taking the Laplace transform

$$\left[s\underline{X} - \underline{x}(0)\right] = A\underline{X} + B\underline{U}$$

$$\left[sI\underline{X} - \underline{x}(0)\right] = A\underline{X} + B\underline{U}$$

$$\left[sI - A\right]\underline{X} = \underline{x}(0) + B\underline{U}$$

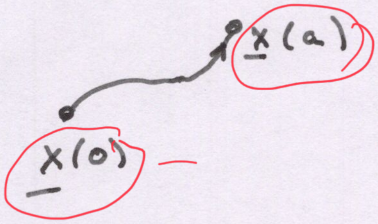
$$\underline{X} = \left[sI-A\right]^{-1}\underline{x}(0) + \left[sI-A\right]^{-1}B\underline{U}$$

$$\boxed{\left[sI-A\right]^{-1} \longleftrightarrow \Phi}$$ L-trans pair

Invert

$$\underline{x} = \Phi\,\underline{x}(0) + \Phi * B\underline{u}$$

Controllable



Controllable ~~if~~

if ~~on~~ can start at any $\underline{x}(0)$ a travel to any locatin $\underline{x}(a)$ under the action of a input $\underline{u}$

How can one determine if a system is controllable?

given $\dot{\underline{x}} = A\underline{x} + B\underline{u}$
where A has unique eigenvalues

let $\underline{x} = E\underline{z}$

$$E\dot{\underline{z}} = AE\underline{z} + B\underline{u}$$

$$\dot{\underline{z}} = E^{-1}AE\underline{z} + E^{-1}B\underline{u}$$

$$\dot{\underline{z}} = \Lambda\underline{z} + E^{-1}B\underline{u}$$

If any row of $E^{-1}B$ is all zero the system is <u>not</u> controllable example

$$\underline{\dot{z}} = \underline{\Lambda}\,\underline{z} + E^{-1}B\,\underline{u}$$

①
$$\begin{bmatrix}\dot{z}_1\\ \dot{z}_2\end{bmatrix} = \begin{bmatrix}\lambda_1 & 0\\ 0 & \lambda_2\end{bmatrix}\begin{bmatrix}z_1\\ z_2\end{bmatrix} + \underbrace{\begin{bmatrix}0 & 0 & 0\\ 6 & 1 & 7\end{bmatrix}}\begin{bmatrix}u_1\\ u_2\\ u_3\end{bmatrix}$$

note $\begin{bmatrix}0 & 0 & 0\\ 6 & 1 & 7\end{bmatrix}$ ← all zero not controllable

not function $\underline{u}$

$$\dot{z}_1 = \lambda_1 z_1$$
$$\dot{z}_2 = \lambda_2 z_2 + 6u_1 + 1u_2 + 7u_3$$

②
$$\underline{\dot{z}} = \underline{\Lambda}\,\underline{z} + \begin{bmatrix}a & 0 & b\\ 0 & c & d\\ q & 1 & r\end{bmatrix} u_1$$

not controllable if

$a = b = 0$

~~or~~

or

$c = d = 0$

Observability

$$\underline{\dot{x}} = A\underline{x}$$

$$\underline{y} = C\underline{x}$$

let $E\underline{z} = \underline{x}$

$$\underline{\dot{z}} = \Lambda \underline{z}$$

$$\underline{y} = (CE)\underline{z}$$

if any column of CE is all zero the system is not obs.

Observable

If on can ~~be~~ determine $\underline{x}(0)$ from observation of $\underline{y}$ over a finite interval of time example

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$$

$$\dot{z}_1 = \lambda z_1$$

$$z_1(t) = e^{\lambda t} z_1(0)$$

$$\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} e^{\lambda_1 t} z_1(0) \\ e^{\lambda_2 t} z_2(0) \end{bmatrix}$$

CE

$$\underline{y} = \begin{bmatrix} 0 & 1 \\ 0 & 2 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} \Rightarrow \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} z_2 \\ 2 z_2 \\ 3 z_2 \end{bmatrix}$$

not obs

$y_1 = z_2$

$y_2 = z_2 2$

$y_3 = z_2 3$

note z_1 is not obs by examining y_1, y_2, y_3

example

$$\dot{\underline{z}} = \Lambda \underline{z}$$

$$\underline{y} = (CE)\underline{z}$$

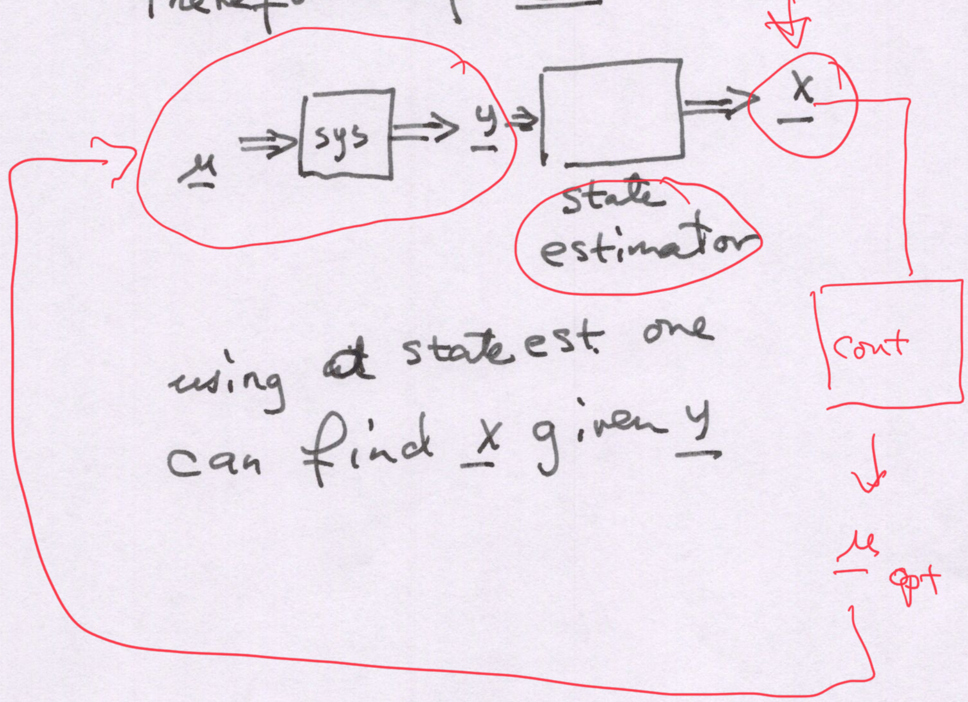
$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} \Rightarrow \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} z_1(0)e^{\lambda_1 t} \\ z_2(0)e^{\lambda_2 t} \end{bmatrix}$$

not obs

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 \\ 0 & 2 \\ 0 & 3 \end{bmatrix}}_{CE} \begin{bmatrix} z_1(t) \\ z_2(t) \end{bmatrix} \Rightarrow \begin{aligned} y_1 &= z_2(t) \\ y_2 &= 2z_2 \\ y_3 &= 3z_2 \end{aligned}$$

z_1 is not obs at output

therefore if obs



using state est one can find $\underline{x}$ given $\underline{y}$

$$\dot{\underline{x}} = A\underline{x} + B\underline{u}$$

$$\underline{y} = C\underline{x}$$

if <u>controllable</u> given $\underline{x}$ one can

find $\underline{u}$ using control action

ie