/

1a

$$x(n) = (3n-1)\,u(-n-1) + 3^{n-1}u(n-1)$$

$$X(z) = 3\sum_{n=-1}^{-\infty} n z^{-n} - \sum_{n=-1}^{-\infty} z^{-n} + \sum_{n=1}^{\infty}\left(\frac{3}{z}\right)^n \frac{1}{3}$$

$m=-n$ $\quad$ $m=-n$

$$-3\sum_{m=1}^{\infty} m z^{m} + \sum_{m=1}^{\infty} z^{m} + \frac{1}{3}\sum_{n=1}^{\infty}\left(\frac{3}{z}\right)^n$$

$$-z3\frac{d}{dz}\left[\frac{1}{1-z}\right] + \frac{1-}{1-z} + \frac{1}{3}\frac{\frac{3}{z}}{1-\frac{3}{z}}$$

$\cancel{-3z\sum_{m=}}$

$|z|<1 \qquad \underbrace{|z|<1 \qquad \left|\frac{3}{z}\right|<1}_{\text{no intersection}}$

(b) $$\sum_{n=0}^{-\infty}\left(\frac{1}{2z}\right)^n + \sum_{n=0}^{\infty}\frac{e^{jn\pi/10}}{2}z^{-n} + \sum_{n=0}^{\infty}\frac{e^{-jn\pi/10}}{2}z^{-n}$$

$m=-n$

$$\sum_{m=0}^{\infty}(2z)^m + \frac{1}{2}\sum_{n=0}^{\infty}\left(\frac{e^{j\pi/10}}{z}\right)^n + \frac{1}{2}\sum_{n=0}^{\infty}\left(\frac{e^{-j\pi/10}}{z}\right)^n$$

$$\frac{1-}{1-2z} + \frac{1}{2}\frac{1}{1-\frac{e^{j\pi/10}}{z}} + \frac{1}{2}\frac{1}{1-\frac{e^{-j\pi/10}}{z}}$$

$\underbrace{|2z|<1 \qquad \left|\frac{e^{j\pi/10}}{z}\right|>1}_{\text{no intersection}} \qquad \left|\frac{1}{z}\right|>1$

1.c

$\delta(n+2k) \neq 0$ $\qquad$ $n = -2k$ given $k \geq 0$

$\underline{2k = -n}$ $\rightarrow$ $\underline{n \text{ even}}$

$$x(n) = \begin{cases} a^{-n} & n \geq 0 \text{ \& even} \\ 0 & n \geq 0 \text{ \& odd} \end{cases}$$

$$X(z) = \cancel{\sum_{n=0}^{\infty} a^{-2n}}\, z^{-n} \qquad \sum_{\substack{n=0 \\ n \text{ even}}}^{\infty} a^{-n} z^{-n}$$

$\underline{n = 2m}$

$$X(z) = \sum_{m=0}^{\infty} (az)^{-2m} = \frac{1}{1 - \frac{1}{(az)^2}}$$

$$\frac{1}{|az|} < 1 \Rightarrow |z| > \frac{1}{|a|}$$

2

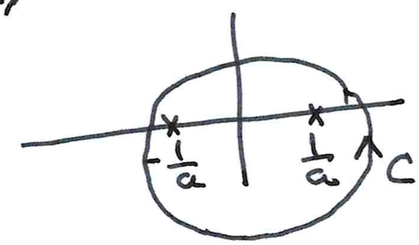
$$X = \frac{z^2}{z^2 - \frac{1}{a^2}} = \frac{z^2}{\left(z+\frac{1}{a}\right)\left(z-\frac{1}{a}\right)} \qquad |z| > \frac{1}{|a|}$$

$$x(n) = \frac{1}{2\pi i} \oint_C \frac{z^{n+1}}{\left(z+\frac{1}{a}\right)\left(z-\frac{1}{a}\right)}\, dz$$

$n+1 \geqslant 0$

$n+1 < 0$

$x(n) = 0$



$$x(n) = \left.\frac{z^{n+1}}{z-\frac{1}{a}}\right|_{z=-\frac{1}{a}} + \left.\frac{z^{n+1}}{z+\frac{1}{a}}\right|_{z=\frac{1}{a}}$$

$$x(n) = \frac{\left(-\frac{1}{a}\right)^{n+1}}{-\frac{2}{a}} + \frac{\left(\frac{1}{a}\right)^{n+1}}{\frac{2}{a}}$$

$$x(n) = \frac{1}{2}\left[\left(-\frac{1}{a}\right)^n + \left(\frac{1}{a}\right)^n\right]$$

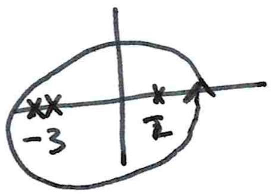
$$x(n) = \frac{1}{2}\left(\frac{1}{a}\right)^n \left[(-1)^n + 1\right]$$

3. $H(z) = 3z^3 + 2z^2 + z$

$$G(z) = \frac{z}{(z+3)^2(z-2)} \qquad h(n) = 3g(n+2) + 2g(n+1) + g$$

$$g(n) = \frac{1}{2\pi i} \oint_c \frac{z^n}{[z+3]^2[z-2]}$$

(a)



$|z| > 3$

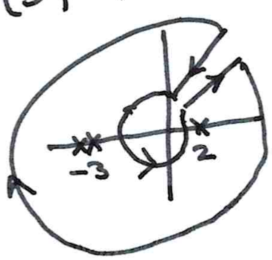
$n \geq 0$

$$g(n) = \frac{1}{1!} \frac{d}{dz}\left[\frac{z^n}{z-2}\right]\Bigg|_{z=-3} + \frac{z^n}{[z+3]^2}\Bigg|_{z=2}$$

$n < 0$

$$g(n) = 0$$

(b) $|z| < 2$



$n \geq 0$

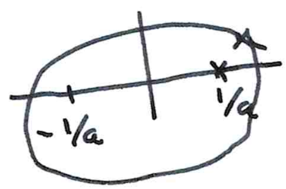
$$g(n) = 0$$

$n < 0$

$$g(n) = (-1)\frac{1}{1!} \frac{d}{dz}\left[\frac{z^n}{z-2}\right]\Bigg|_{z=-3} + (-1)\frac{z^n}{[z+3]^2}\Bigg|_{z=2}$$

(2) $$\mathbb{X}(z) = \frac{1}{1-a^2z^2} = \frac{\frac{1}{a^2}}{\frac{1}{a^2}-z^2} = -\frac{1}{a^2}\,\frac{1}{z^2-\frac{1}{a^2}}$$

$$\mathbb{X}(z) = \frac{1}{a^2}\,\frac{1}{(z+\frac{1}{a})(z-1/a)} \qquad |z| > \left|\frac{1}{a}\right|$$



$n-1 \geq 0$

$$x(n) = \frac{1}{2\pi i}\oint \frac{z^{n-1}}{a^2(z+1/a)(z-1/a)}\,dz$$

$$x(n) = \frac{1}{a^2}\left[\frac{\left(-\frac{1}{a}\right)^{n-1}}{(-2/a)} + \frac{\left(\frac{1}{a}\right)^{n-1}}{(2/a)}\right]$$

$$\cancel{\frac{1}{a^2}\left[\frac{a}{2}\right.}$$

$$\frac{1}{a^2}\,\frac{1}{2}\left[-(-1)^n+1\right]\left(\frac{1}{a}\right)^n$$

6

4

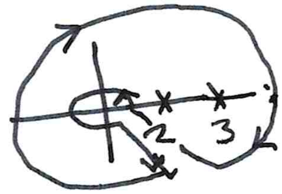
(a) $x(n) = 3^n u(-n) * 2^n u(-n)$

$$A(z) = \sum_{n=-\infty}^{0} \left(\frac{3}{z}\right)^n \underset{m=-n}{\Longrightarrow} \sum_{m=0}^{\infty} \left(\frac{z}{3}\right)^m = \frac{1-\left(\frac{z}{3}\right)^{\infty+1}}{1-\left(\frac{z}{3}\right)} \quad |z|<3$$

$$B(z) = \sum_{n=-\infty}^{0} \left(\frac{2}{z}\right)^n \underset{m=-n}{\Longrightarrow} \sum_{m=0}^{\infty} \left(\frac{z}{2}\right)^m = \frac{1-\left(\frac{z}{2}\right)^{\infty+1}}{1-\left(\frac{z}{2}\right)} \quad |z|<2$$

$\text{ROC}_x \quad |z|<2$

$$X(z) = \left(\frac{1}{1-z/3}\right)\left(\frac{1}{1-z/2}\right) = \frac{6}{(z-3)(z-2)}$$



$$x(n) = \frac{1}{2\pi i} \oint \frac{z^{n-1}6}{[z-3][z-2]} dz$$

$n-1 \geq 0 \quad x(n)=0$

$n-1 < 0$

$$x(n) = \left.\frac{z^{n-1}6}{z-3}\right|_{z=2}(-1) + \left.\frac{z^{n-1}6}{z-2}\right|_{z=3}$$

4b

$$A(z) = \sum_{n=0}^{\infty} 0.6^{n} z^{-n} + \sum_{n=-1}^{-\infty} 0.6^{-n} z^{-n}$$

$$= \sum_{n=0}^{\infty} \left(\frac{0.6}{z}\right)^{n} + \sum_{n=-1}^{-\infty} \left(\frac{1}{0.6z}\right)^{n}$$

$m=-1$

$$= \sum_{n=0}^{\infty} \left[\frac{0.6}{z}\right]^{n} + \sum_{m=1}^{\infty} (0.6z)^{m}$$

$$= \frac{1}{1-\frac{0.6}{z}} + \frac{0.6z - 0}{1-0.6z}$$

$$\left|\frac{0.6}{z}\right| < 1 \cap |0.6z| < 1 \Rightarrow 0.6 < |z| < \frac{1}{0.6}$$

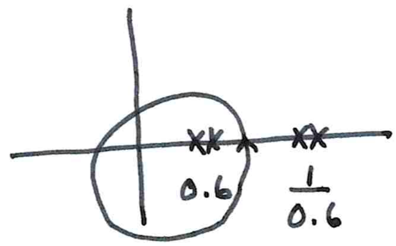
$$A(z) = \frac{-z}{(z-0.6)(z-\frac{1}{0.6})} \qquad 0.6 < |z| < \frac{1}{0.6}$$

$$X = A(z)\,A\left(\frac{1}{z}\right) \qquad 0.6 < \left|\frac{1}{z}\right| < \frac{1}{0.6}$$

$$A\left(\frac{1}{z}\right) = \frac{-1/z}{(\frac{1}{z}-0.6)(\frac{1}{z}-\frac{1}{0.6})}$$

$$A\left(\frac{1}{z}\right) = \frac{-z}{(z-0.6)(z-\frac{1}{0.6})}$$

$$X = \frac{z^{2}}{(z-0.6)^{2}(z-\frac{1}{0.6})^{2}} \qquad 0.6 < |z| < \frac{1}{0.6}$$



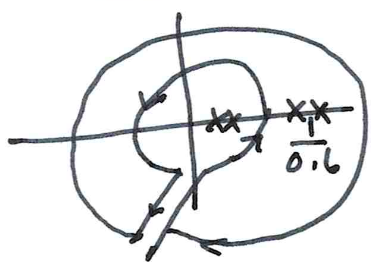
$n+1 \geq 0$

$$x(n) = \oint \frac{1}{2\pi i} \frac{z^{n+1}}{(z-0.6)^2\left(z-\frac{1}{0.6}\right)^2} dz$$

$$\cancel{x(n) = z^{n+1}}$$

$$x(n) = \frac{d}{dz}\left[\frac{z^{n+1}}{\left(z-\frac{1}{0.6}\right)^2}\right]\Bigg|_{z=0.6}$$

$n+1 < 0$



$$x(n) = \frac{1}{1!}\frac{d}{dz}\left[\frac{z^{n+1}}{(z-0.6)^2}\right]\Bigg|_{z=\frac{1}{0.6}} (-1)$$

5

(c) $z = e^{sT}$

$$\hat{H}(z = e^{sT}) = -\left[\frac{1}{1 - e^{(s-3)T}}\right] + \left[\frac{1}{1 - e^{(s-1)T}}\right]$$

$$1 - e^{(s-3)T} = 1 - \left\{ 1 + (s-3)T + \frac{[(s-3)T]^2}{2!} + \cdots \right\}$$

$$= -\left\{ (s-3)T + \frac{[(s-3)T]^2}{2!} + \cdots \right\}$$

$$= -(s-3)T \left\{ 1 + \frac{(s-3)T}{2!} \cdots \right\}$$

$$1 - e^{(s-1)T} = -(s-1)T \left\{ 1 + \frac{(s-1)T}{2!} \cdots \right\}$$

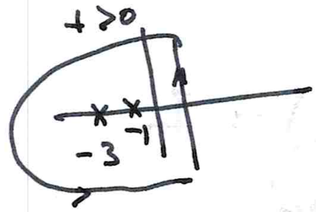
$$\hat{H} = \frac{1}{(s-3)T\left\{1 + \frac{(s-3)T}{2!} \cdots\right\}} - \frac{1}{(s-1)T\left\{1 + \frac{(s-1)T}{2!} + \cdots\right\}}$$

5.

$t<0$

$h(t)=0$

(a)



$t\geq 0$

$$h(t)= \frac{2}{s+1} e^{st}\Big|_{s=-3} + \frac{2}{s+3} e^{st}\Big|_{s=-1}$$

$$h(t) = \left[-e^{-3t} + e^{-t}\right] u(t)$$

$$h(nT) = \left[-\left(e^{-3T}\right)^n + \left(e^{-T}\right)^n\right]$$

(b)

$$\hat{H}(z) = \sum_{n=0}^{\infty} -\left(\frac{e^{-3T}}{z}\right)^n + \left(\frac{e^{-T}}{z}\right)^n$$

$$= -\left[\frac{1}{1-\frac{e^{-3T}}{z}}\right] + \left[\frac{1}{1-\left(\frac{e^{-T}}{z}\right)}\right]$$

$$\left|\frac{e^{-3T}}{z}\right|<1 \cap \left|\frac{e^{-T}}{z}\right|<1$$

$$|z| > e^{-T}$$

if

$$\left|\frac{|s-3|T}{2}\right| \ll 1$$

$$\left|\frac{|s-1|}{2}T\right| \ll 1$$

then

$$\hat{H} \simeq \frac{1}{(s-3)T} - \frac{1}{(s-1)T} = \frac{1}{T}\left[\frac{2}{(s-3)(s-1)}\right]$$

let s = 0 in the condition

$$\left|\frac{(s-3)T}{2}\right| \ll 1$$

$$\left|\frac{3}{2}T\right| \ll 1 \quad @ s=0$$

using $\frac{1}{10}$ goal

T	$\lvert\frac{3}{2}T\rvert$
$\frac{1}{6}$	$\frac{1}{4}$
$\frac{1}{12}$	$\frac{1}{8}$
$\frac{1}{24}$	$\frac{1}{16}$ ✓