1\. a

$$\mathbb{X} = \int_0^{\infty} t^3 e^{-[s+6]t}\,dt + \int_{-\infty}^{0} t\, e^{-st}\,dt$$

$$\left.\frac{-6\,e^{-[s+6]t}}{(s+6)^4}\right|_0^{\infty} \qquad \left.-\frac{e^{-st}}{s^2}\right|_{-\infty}^{0} \Rightarrow \boxed{\mathbb{X} = \frac{1}{s^2}(-1) + \frac{6}{(s+6)^4}}$$

$Re(s+6)>0 \cap Re(s)<0$

ROC_x $\boxed{-6 < Re(s) < 0}$

b

$$\mathbb{X}(s) = \int_{-\infty}^{0} \left(\tfrac{1}{2}\right) e^{-[s-j\omega]t+j\phi} + \left(\tfrac{1}{2}\right) e^{-[s+j\omega]t-j\phi}\,dt$$

$$= \left(\tfrac{1}{2}\right)\left[\frac{e^{-[s-j\omega]+j\phi}}{-[s-j\omega]} + \frac{e^{-[s+j\omega]-j\phi}}{-[s+j\omega]}\right]\Bigg|_{-\infty}^{0}$$

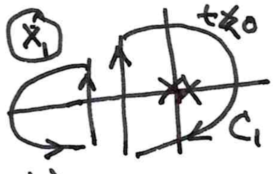
$$\boxed{\mathbb{X}(s) = -\frac{1}{2}\left[\frac{e^{j\phi}}{s-j\omega} + \frac{e^{-j\phi}}{s+j\omega}\right] \quad Re(s)<0}$$

c.

$$\mathbb{X}(s) = \int_{-1}^{\infty} e^{-[s+a]t}\,dt = \left.\frac{e^{-[s+a]t}}{-[s+a]}\right|_{-1}^{\infty} = \boxed{\frac{e^{+[s+a]}}{[s+a]} \quad Re(s) > -Re(a)}$$

$Re(s+a)>0$

2 (a) $x = x_1 + x_2$



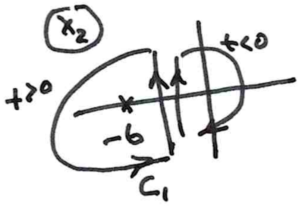
$t<0$

$$x_1 = \frac{1}{2\pi i} \oint_{C_1} \frac{-e^{+st}}{s^2}\, ds = \frac{1}{1!}(-1)\frac{d}{ds}\left[-e^{st}\right]\Big|_{s=0}$$

$$\boxed{x_1 = t}$$

$t>0$

$$\boxed{x_1 = 0}$$



$t>0$

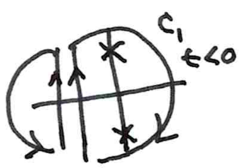
$$x_2 = \frac{1}{2\pi i} \oint_{C_1} \frac{6e^{st}}{[s+6]^4}\, ds = \frac{1}{3!}\frac{d^3}{ds^3}\left[6e^{st}\right]\Big|_{s=-6}$$

$$\boxed{x_2 = t^3 e^{-6t}}$$

$t<0$

$$\boxed{x_2 = 0}$$

(b)



$t<0$

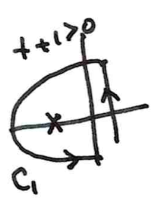
$$x = \frac{1}{2\pi i} \oint_{C_1} \left(-\frac{1}{2}\right)\left[\frac{e^{j\phi}}{s-j\omega} + \frac{e^{-j\phi}}{s+j\omega}\right] e^{st}\, ds$$

$$= (-1)\left(-\frac{1}{2}\right)\left[e^{st}e^{j\phi}\Big|_{s=j\omega} + e^{st}e^{-j\phi}\Big|_{s=-j\omega} \right]$$

$$\boxed{x = \cos(\omega t + \phi)}$$

$t>0$ $\boxed{x = 0}$

(c)



$t+1>0$

$$x = \frac{1}{2\pi i} \oint_{C} \frac{e^{s(t+1)}e^{a}}{s+a}\, ds = e^{s(t+1)}e^{a}\Big|_{s=-a} \Rightarrow \boxed{x = e^{-at}}$$

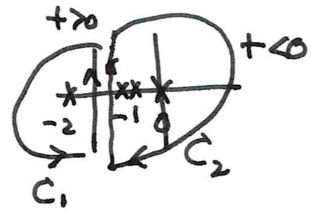
$t+1>0$!

$t+1<0$

$$\boxed{x = 0}$$

3 (a) $\underline{t>0}$

$$x(t) = \frac{1}{2\pi i} \oint_{C_1} X(s)\, e^{st}\, ds$$

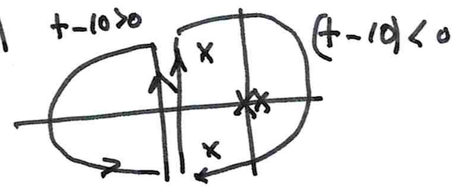


$$x(t) = \frac{e^{st}}{(s+1)s} \Bigg|_{s=-2}$$

$\underline{t<0}$

$$x(t) = (-1)\frac{e^{st}}{(s+2)(s+1)} \Bigg|_{s=0} + (-1)\frac{e^{st}}{s(s+2)} \Bigg|_{s=1}$$

(b)



$\underline{t-10>0}$

$x(t)=0$

$\underline{(t-10)<0}$

$$x = (-1)\frac{1}{1!}\frac{d}{ds}\left[\frac{e^{s(t-10)}(s+2)}{(s+1+i)(s+1-i)}\right]\Bigg|_{s=0} + (-1)\left[\frac{e^{s(t-10)}(s+2)}{s^2(s+1+i)}\right]\Bigg|_{s=-1+i}$$

$$(-1)\left[\frac{e^{s(t-10)}(s+2)}{s^2(s+1-i)}\right]\Bigg|_{s=-1-i}$$

4. $$\int_0^\infty \left[\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 5x\right] e^{-st}\,dt = \int_0^\infty 3e^{(s-1)t}\,dt$$

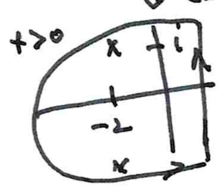
$$X[s^2+4s+5] = x(0)[s+4] + x'(0) + \frac{3}{s+1}$$

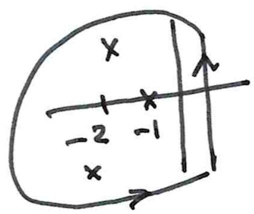
$$X = \frac{x(0)[s+4]+x'(0)}{(s+2+i)(s+2-i)} + \frac{3}{(s+1)(s+2+i)(s+2-i)}$$

$\Downarrow$ causal

causal

ROC chosen s.t. x is causal





$\cancel{x(t) = e^{st}[x(0)[s+4]+x'(0)]}\ \{$

t>0

$$x(t) = X e^{st}(s+2+i)\Big|_{s=-2-i} + X e^{st}(s+2-i)\Big|_{s=-2+i}$$

$$+ \frac{3e^{st}}{(s^2+4s+5)}\Big|_{s=-1}$$

t<0

$x(t)=0$

4b

$$\bar{X}s - x(0) + 6\bar{X} = \frac{1}{s+6}$$

$$\bar{X} = \overset{x_1}{\frac{x(0)}{s+6}} + \overset{x_2}{\frac{1}{(s+6)^2}}$$

$t>0$

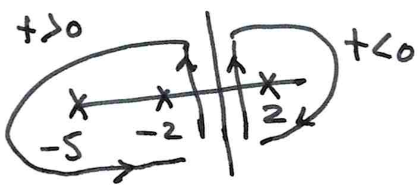
$$x_1 = x(0)e^{st}\Big|_{s=-6} \;; \quad x_2 = \frac{1}{1!}\frac{d}{ds}e^{st}\Big|_{s=-6}$$

$$\boxed{x = x(0)e^{-6t} + te^{-6t}}$$

5

$$Y = XH = \frac{s}{(s+2)(s-2)(s+5)} \qquad \mathrm{ROC}_y = \mathrm{ROC}_x \cap \mathrm{ROC}_y$$

$$-2 < \mathrm{Re}(s) < 2$$



<u>$t>0$</u>

$$y = \left.\frac{s e^{st}}{(s-2)(s+5)}\right|_{s=-2} + \left.\frac{s e^{st}}{(s+2)(s-2)}\right|_{s=-5}$$

<u>$t<0$</u>

$$y = \left.\frac{s e^{st}}{(s+2)(s+5)}\right|_{s=2} (-1)$$