University of Massachusetts Lowell
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EECE 5090 Linear Systems

1. Find the Laplace transform of the following functions. Please denote their regions of convergence.
a. $x(t) = t^3 e^{-6t} u(t) + tu(-t)$
b. $x(t) = \cos(\omega t + \phi) u(-t)$
c. $x(t) = e^{-at} u(t+1)$

2. Invert the transforms obtained in Problem 1 using contour integration. Please denote the contour used.

3. Find the inverse of the Laplace transforms given using contour integration.

a. $X(s) = \dfrac{s+1}{s(s+1)^2(s+2)} \quad -2 < Re(s) < -1$

b. $X(s) = \dfrac{e^{-10s}(s+2)}{s^2(s+1-i)(s+1+i)} \quad Re(s) < -1$

4. Solve the following differential equations using the Laplace transform. The initial conditions are arbitrary and the resulting time signal is causal.

a. $\dfrac{d^2x}{dt^2} + 4\dfrac{dx}{dt} + 5x = 3e^{-t}u(t)$

b. $\dfrac{dx}{dt} + 6x = e^{-6t}u(t)$

5. Consider the input $x(t)$ to a linear-time-invariant system has the Laplace transform

$$X(s) = \frac{1}{s+5} \qquad Re(s) > -5$$

The Laplace transform of the impulse response $h(t)$ is

$$H(s) = \frac{s}{(s+2)(s-2)} \qquad -2 < Re(s) < 2$$

a. Determine the output $y(t) = x(t) * h(t)$.

b. Determine the correlation $R(t)$ of $x(t)$ and $h(t)$.