University of Massachusetts Lowell
Department of Electrical and Computer Engineering
16.520 Computer Aided Engineering Analysis
Problem Set 3

1. Consider the solution of the differential equation

$$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 0$$

for over the interval $0 \le x \le 1$ for the boundary conditions where $y(0) = 1.0$ and $y(1) = 2.0$. The approach to solution will be based on you spatially discretizing the domain and approximating the derivative operators. Define $x_j = j\delta x$, $y(x_j) = y_j$ and $\delta x = 1/N$ where $j = (0, N)$. At x_j

$$\frac{d^2y}{dx^2} = \frac{y_{j-1} - 2y_j + y_{j+1}}{(\delta x)^2}$$

$$\frac{dy}{dx} = \frac{y_{j+1} - y_{j-1}}{(2\delta x)}$$

a. Analytically determine the exact solution.

b. Find y_j for $N = 10$ using a direct solution method.

c. Find the solution y_j using the Jacobi and the Gauss-Siedel methods.

d. Compare the rate of convergence and accuracy of the methods.

2. Consider the transfer function for the SISO linear time invariant system where the input is given by its Laplace transform $U(s)$ and output $Y(s)$

$$\frac{Y}{U} = \frac{20(s+10)(s+6)}{(s+1)(s+2)(s+3)}$$

a. Using the phase variable decomposition find the the eigenvalues and eigenvectors of the state-matrix.

b. Find the state-transition matrix of the system

c. Given that $u(t) = \delta(t)$ find the forced response of the system.

3. Given the system $B d\underline{x}/dt = A\underline{x}$ where $\varepsilon = 1 \times 10^{-4}$ and

$$A = \begin{bmatrix} -2 & 1 & 2 \\ 2 & 3 & -2 \\ 1 & -2 & 3 \end{bmatrix} \quad B = \begin{bmatrix} -2 & 2 & 4 \\ 3 & 1 & -1 \\ 0 & 0 & \varepsilon \end{bmatrix}$$

a. Determine the eigenvalues and eigenvectors of the system without inverting the B matrix.

b. Determine the state-transition matrix.

4. The following US Census data is to be modeled by the polynomial

$$P(y) = c_1 + c_2 y + c_3 y^2$$

where the variable y is the year.

Year	Population
1900	75,994,575
1910	91,972,266
1920	105,710,620
1930	122,775,046
1940	131,669,275
1950	150,697,361
1960	179,323,175
1970	203,235,298
1980	227,224,681
1990	249,438,712
2000	281,421,906

a. Scale the year by 10 and the population by 100,000.

b. Using data from years 1900 to 1980 determine the values of c_n using SVD least-squares.

c. Compare your results after removing the contribution of the smallest singular value.

d. How well does your model match the population given for 1990 and 2000. State the error and plot using gnuplot displaying the data as points and the model curve as a solid line.