Department of Electrical and Computer Engineering

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1. Given the state equations

$$\begin{bmatrix} x'_1 \\ x'_2 \\ x'_3 \end{bmatrix} = \begin{bmatrix} 1/2 & -3/2 & -3/2 \\ -1 & 0 & -1 \\ -1/2 & 1/2 & 3/2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} a & b \\ 0 & 0 \\ 1 & c \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$\underline{y} = \begin{bmatrix} g & 1 & 4 \\ 0 & 0 & 1 \\ 0 & 3 & -1 \end{bmatrix} \underline{x}$$

   a. Determine the eigenvalues and eigenvectors of the system matrix.
   b. Determine the state-transition matrix
   c. Determine the conditions for the system to be controllable
   d. Determine the conditions for the system to be observable

2. Consider the uncontrolled system model as

$$\dot{x} = x + u$$

The performance is measured over time interval (0,1) as

$$PI = \int_0^1 \left[x^2/2 + u^2/2 \right] dt$$

   a. Determine the state-function of Pontryagin H.
   b. Determine the optimal input u^o.
   c. Determine the equations governing the controlled system in terms of x and λ.
   d. Evaluate the general solution of the equations in part(c).
   e. Find $u^o(t)$ for the boundary conditions $x(0) = 1$ and $x(1) = 2$.

3. Consider the system

$$\underline{\dot{x}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 2 & 1 & -2 \end{bmatrix} \underline{x} + \begin{bmatrix} 3a & 3b \\ 3 & 3 \\ 1 & -1 \end{bmatrix} \underline{u}$$

$$\underline{y} = \begin{bmatrix} \alpha & 1 & 1 \\ 2 & \beta & 1 \end{bmatrix} \underline{x}$$

   a. Find the eigenvalues and eigenvectors of the system.
   b. Find the state transition matrix.

c. Determine when the system is controllable.

d. Determine when the system is observable.

4. Consider the transfer function for a causal system

$$\frac{Y}{U} = \frac{10(s+4)}{s^3 + 9s^2 + 23s + 15}$$

where U and Y are the Laplace transform of the input and output respectively. Using phase variables representation of the system matrices (A, B, C) where $\dot{\underline{x}} = A\underline{x} + B\underline{u}$ and $\underline{y} = C\underline{x}$.

a. Determine the state and output equations in terms of the state vector $\underline{x}$, input u and output y.

b. Find the state transition matrix.

c. If $u(t) = \delta(t)$ determine $y(t)$ using the STM.

d. If $u(t)$ is equal to the unit step function what is $y(t)$. Explain your answer.

5. Consider the system

$$\dot{\underline{x}} = \begin{bmatrix} 2 & 3 & 1 \\ 0 & -1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \underline{x}$$

a. Find the STM and general solution for the arbitary initial condition $\underline{x}(0)$

b. Given the initial condition $x_3(0) = 0$ and terminal conditions $x_1(1) = 1$ and $x_2(1) = 2$ determine the required initial conditions $x_1(0)$ and $x_2(0)$.