

(b)  $1+GH=0 \Rightarrow 1 + \left(\frac{4s+k}{s^2}\right)\left(\frac{1}{s+2}\right) = 0 \Rightarrow s^3 + 2s^2 + 4s + k = 0$

(c)

$s^3$	1	4
$s^2$	2	k
$s$	$\frac{8-k}{2}$	
$s^0$	k	

Stable  $\rightarrow 8-k > 0$

$\rightarrow k > 0$

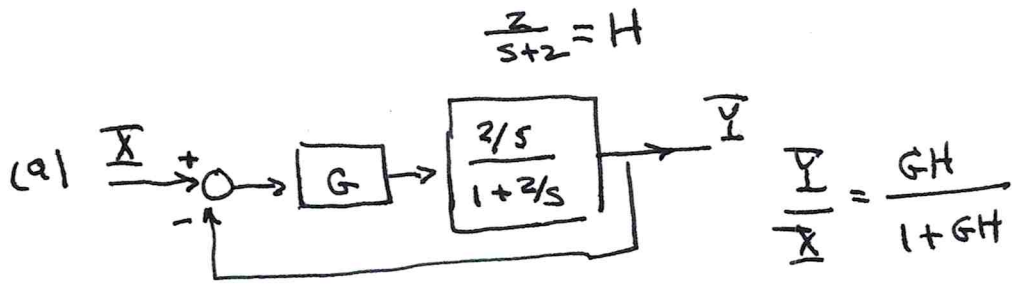
Stable if  $8 > k > 0$

(d) Marg. stable

(k=8) aux eqn:  $2s^2 + k = 0 \Rightarrow s^2 + \frac{8}{2} = 0$

$s = \pm j2 = j\omega$   
 $\omega = 2$  rad/sec  
 $f = \frac{2}{2\pi} \text{ Hz}$

2



$$(b) \quad E = X - Y \Rightarrow \frac{E}{X} = 1 - \frac{Y}{X} = \frac{1}{1+GH}$$

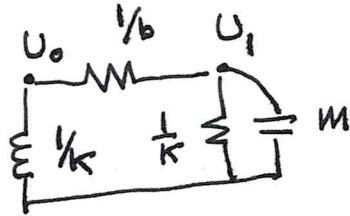
$$X = \frac{1}{s^2}; \quad E = \left( \frac{1}{s^2} \right) \left[ \frac{1}{1 + \frac{GZ}{s+2}} \right]$$

$$e(\infty) = \lim_{s \rightarrow 0} s \frac{1}{s + 2sG} \frac{1}{s+2} = \frac{1}{2} \Rightarrow \lim_{s \rightarrow 0} \frac{1}{sG} = \frac{1}{2}$$

$$\boxed{G = \frac{k}{s} \Rightarrow k = 2}$$

3.

(a)



$$\frac{U_1}{U_0} = \frac{\frac{3}{k} \parallel \frac{1}{ms}}{\frac{1}{b} + \frac{3}{k} \parallel \frac{1}{ms}}$$

$$\frac{3}{k} \parallel \frac{1}{ms} = \left( \frac{k}{s} + ms \right)^{-1} = \left( \frac{k + ms^2}{s} \right)^{-1} = \frac{s}{k + ms^2}$$

$$\frac{U_1}{U_0} = \frac{sb}{ms^2 + bs + k} = \frac{s \frac{1}{2}}{s^2 + s/2 + 1}$$

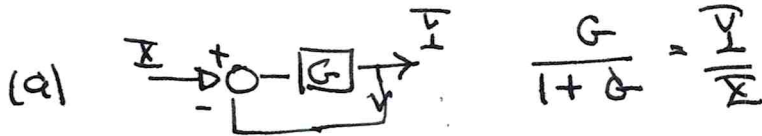
$$\delta \leftrightarrow U_1 = \frac{1}{2} \left( \frac{s}{s^2 + s/2 + 1} \right)$$

$$\underline{t > 0} \quad u_1 = \frac{e^{-t/4}}{2} \left\{ \cos \left[ \frac{\sqrt{15}}{4} t \right] - \sin \left[ \frac{\sqrt{15}}{4} t \right] \sqrt{\frac{1}{15}} \right\}$$

 $t < 0$ 

$$u_1(t) = 0$$

4.



$$1+G=0 \Rightarrow 1 + \frac{ks(s+2)}{(s^2-4s+8)(s+3)} = 0$$

$$s^3 + (k-1)s^2 + (2k-4)s + 24 = 0$$

$s^3$	1	$2k-4$	stable $(k-1) > 0 \Rightarrow \boxed{k > 1}$
$s^2$	$(k-1)$	24	
$s^1$	$\frac{(k-1)(2k-4)-24}{(k-1)}$		$\rightarrow \frac{2k^2-6k-20}{(k-1)} > 0$
$s^0$	24		analyze $2[k^2-3k-10] > 0$ $2[(k-\frac{3}{2})^2 - (\frac{3}{2})^2 - 10] > 0$ $(k-\frac{3}{2})^2 > (\frac{3}{2})^2 + 10$ $k-\frac{3}{2} > \sqrt{(\frac{3}{2})^2 + 10}$ $\boxed{k > \frac{3}{2} + \sqrt{(\frac{3}{2})^2 + 10}}$

(b)  $k=5$   
aux. poly:

$$(k-1)s^2 + 24 = 0$$

$$4s^2 + 24 = 0$$

$$s = \pm j\sqrt{6}$$

$$\omega = \sqrt{6}$$

$$f = \frac{\sqrt{6}}{2\pi}$$

stable  
if

$$\boxed{k > 5}$$

5.



$$1 + GH = \frac{k(s+2)}{s(s+1)(s+3)} \frac{(s+6)}{(s+7)} + 1 = 0$$

$$s^4 + 11s^3 + (k+31)s^2 + (8k+21)s + 12k = 0$$

$s^4$	1	k+31	12k	
$s^3$	11	8k+21		$k > -\frac{320}{3}$
$s^2$	<del>(11)(k+31) - (8k+21)</del>	12k		$k^2 + \frac{1171}{24}k + \frac{6720}{24} > 0$
	$\frac{3k+320}{11}$			$(k + \frac{1171}{48})^2 - (\frac{1171}{48})^2 + \frac{6720}{24} > 0$
$s^1$	<del>(3k+320)(8k+21) - (11)(12k)</del>			$(k + \frac{1171}{48})^2 > (\frac{1171}{48})^2 - \frac{6720}{24}$
	$\frac{(3k+320)(8k+21) - 132k}{11}$			$k + \frac{1171}{48} > \frac{11}{48} \sqrt{6001}$
	$\frac{24k^2 + 1171k + 6720}{3k+320}$			$k > \frac{11}{48} \sqrt{6001} - \frac{1171}{48} \Rightarrow k > -6.64$
$s^0$	12k			$k > 0$

Stable for  $k > 0$