

$$\begin{aligned}
 \frac{Y}{X} &= \frac{G}{1+GH} \\
 &= \frac{\frac{s+6}{s(s+3)(s+10)}}{1 + \frac{s+6}{s(s+3)(s+10)} H} \\
 &= \frac{s+6}{s(s+3)(s+10) + (s+6)H}
 \end{aligned}$$

b)

$$\begin{aligned}
 \frac{E}{X} &= \frac{1}{1+GH} = \frac{1}{1 + \frac{s+6}{s(s+3)(s+10)} H} \\
 &= \frac{s(s+3)(s+10)}{s(s+3)(s+10) + (s+6)H}
 \end{aligned}$$

c) $X(t) = t u(t) \Rightarrow X(s) = \frac{1}{s^2}$

$$\lim_{t \rightarrow \infty} e(t) = \lim_{s \rightarrow 0} s E(s) \quad \text{final value theorem}$$

$$\begin{aligned}
 \Rightarrow \lim_{t \rightarrow \infty} \frac{de(t)}{dt} &= \lim_{s \rightarrow 0} s^2 E(s) \\
 &= \lim_{s \rightarrow 0} s^2 \frac{E}{X} X(s)
 \end{aligned}$$

$$\lim_{t \rightarrow \infty} \frac{de(t)}{dt} = \lim_{s \rightarrow 0} \frac{s^2}{s^2 + s + \frac{6}{H}} = \frac{1}{1 + \frac{6}{s(s+3)(s+10)H}} = \frac{1}{10} \text{ we want this}$$

$$\Rightarrow \lim_{s \rightarrow 0} 1 + \frac{s+6}{s(s+3)(s+10)H} = 10$$

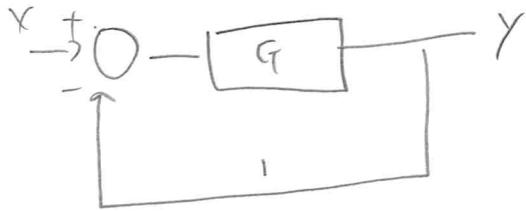
$$\Rightarrow \lim_{s \rightarrow 0} \frac{s+6}{s(s+3)(s+10)H} = 9$$

if $H = s$, then it cancels the s on the bottom, so
 then you have $\frac{6}{(s)(s+3)(s+10)} = \frac{6}{s^3 + 16s^2 + 45s} = \frac{6}{s^2(s+16) + 45} = \frac{6}{s^2(16) + 45} = \frac{6}{256 + 45} = \frac{6}{301} \neq 9$

$$\text{but if } H = 45s, \text{ then } \lim_{s \rightarrow 0} \frac{s+6}{s(s+3)(s+10)} = 45s \\ = \frac{6 \times 45}{3 \times 10} = \frac{270}{30} = 9$$

so H=45s

#2.



$$E = \frac{1}{1+GH} = \frac{1}{1 + \frac{5000}{s(s+75)}}$$

a) $\lim_{t \rightarrow \infty} e(t) = \lim_{s \rightarrow 0} sE(s)$ $x = 5u(t) \Rightarrow x(s) = \frac{5}{s}$

$$= \lim_{s \rightarrow 0} \frac{s^2}{s} \frac{1}{1 + \frac{5000}{s(s+75)}} \quad x = 5t^2 u(t) \Rightarrow x(s) = \frac{5 \cdot 2}{s^3} = \frac{10}{s^3}$$

$$= \lim_{s \rightarrow 0} \frac{s}{1 + \frac{5000}{s(s+75)}}$$

$$= \lim_{s \rightarrow 0} \frac{s}{1 + \frac{5000}{0}} = \frac{s}{1 + \infty} = 0$$

b) $\lim_{t \rightarrow \infty} e(t) = \lim_{s \rightarrow 0} s \frac{10}{s^3} \frac{1}{1 + \frac{5000}{s(s+75)}}$

$$= \lim_{s \rightarrow 0} \frac{10}{s^2} \frac{1}{1 + \frac{5000}{s^2 + 75s}}$$

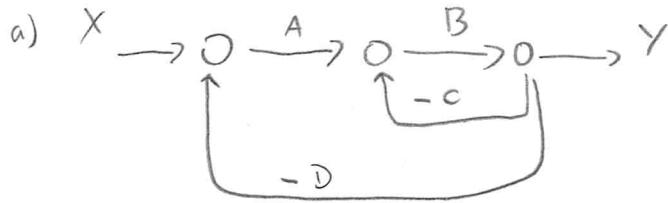
$$= \lim_{s \rightarrow 0} 10 \frac{1}{1 + \frac{5000}{1 + \frac{75s}{s^2}}}$$

$$= \lim_{s \rightarrow 0} 10 \frac{1}{1 + \frac{5000}{1 + \frac{75}{5}}}$$

$$= \lim_{s \rightarrow 0} 10 \frac{1}{1 + \frac{5000}{1 + \infty}}$$

$$= 10 \frac{1}{1+0} = \boxed{10}$$

#3.



b) there are 2 loops:

$$L_1 = -BC$$

$$L_2 = -ABD$$

there is one path:

$$P_1 = AB$$

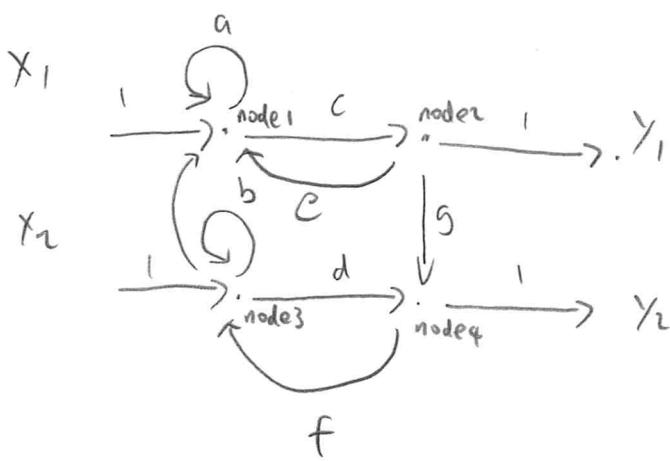
both L_1 and L_2 touch P_1 , so $\Delta_1 = 1$

$$\Delta = 1 - (-BC)(-ABD)$$

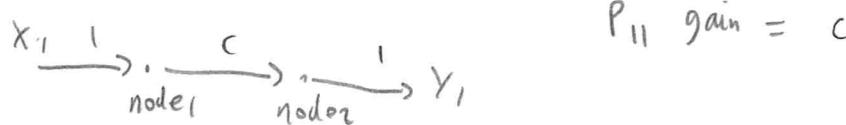
$$= 1 - AB^2CD$$

$$\frac{Y}{X} = \frac{P_1 \Delta_1}{\Delta} = \frac{AB}{1 - AB^2CD}$$

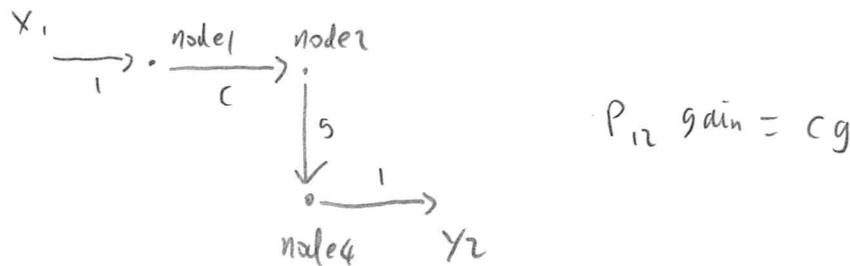
#4.



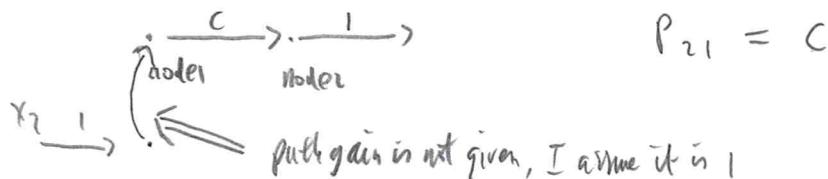
from X_1 to Y_1 , there is one path:



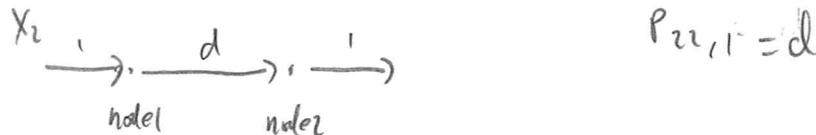
from X_1 to Y_2 there is one path:



from X_2 to Y_1 , there is one path:



from X_2 to Y_2 , there are 2 paths:



Loops :



$$L_1 = a$$



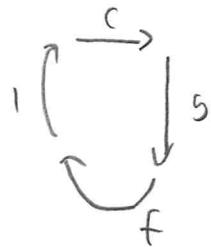
$$L_2 = ce$$



$$L_3 = b$$



$$L_4 = df$$



$$L_5 = cgf$$

a & b do not touch
ce & df do not touch
a, b, ce, df all touch c, g

$$\text{transfer function} = \frac{\text{Path * C-factor}}{\text{determinant}}$$

$$\Delta = 1 - \sum \text{loop gain} + \sum \text{loop gain of any two non-touching loops} \\ - \sum \text{loop gain of any three non-touching loops}$$

$$= 1 - (a + ce + b + df + cgf) + (ab + ce)$$

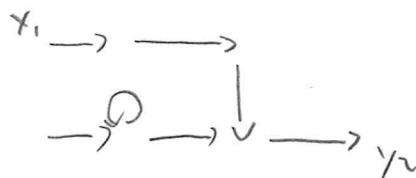
To get P_{11} 's C-factor, remove loops that touch P_{11} , and find the determinant:
So, a, ce, cgf are gone, the signal flow graph is :



$$\Delta_{11} = 1 - (b + df)$$



To get P_{12} 's C-factor, remove a, ce, df, cgf



$$\Delta_{12} = 1 - (b)$$

to get Δ_{11} remove a, b, ce, df, gf, so $\Delta_{11} = 1$

to get Δ_{21} , remove b, df, gf, so $\Delta_{21} = 1 - (a + ce)$

to get $\Delta_{22,1}$ remove b, a, df, ce, gf, so $\Delta_{22,1} = 1$

$$\frac{Y_1}{X_1} = \frac{P_{11} \Delta_{11}}{\Delta} = \frac{c(1-b+df)}{1-(a+ce+b+df+gf)+(ab+ce)}$$

$$\frac{Y_2}{X_1} = \frac{P_{12} \Delta_{12}}{\Delta} = \frac{cg(1-b)}{1-(a+ce+b+df+gf)+(ab+ce)}$$

$$\frac{Y_1}{X_2} = \frac{P_{21} \Delta_{21}}{\Delta} = \frac{c(1)}{1-(a+ce+b+df+gf)+(ab+ce)}$$

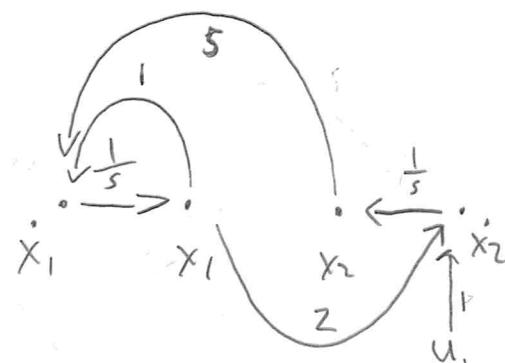
$$\frac{Y_2}{X_2} = \frac{P_{22,1} \Delta_{22,1} + P_{22,2} \Delta_{22,2}}{\Delta} = \frac{d(1-(a+ce)) + cg(1)}{1-(a+ce+b+df+gf)+(ab+ce)}$$

$$A = \begin{bmatrix} \frac{Y_1}{X_1} & \frac{Y_2}{X_1} \\ \frac{Y_1}{X_2} & \frac{Y_2}{X_2} \end{bmatrix}$$

#5

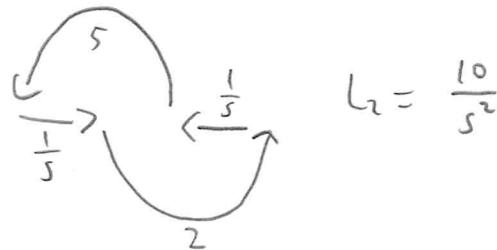
$$a) \quad 5x_1 = x_1 + 5x_2$$

$$5x_2 = 2x_1 + u$$



loops:

$$L_1 = \frac{1}{s}$$



$$\Delta = 1 - (L_1 + L_2) = 1 - \left(\frac{1}{s} + \frac{10}{s^2}\right)$$

path from u to x_1 :

$$P_1 = 1 \left(\frac{1}{s}\right)(5)\left(\frac{1}{s}\right) = \frac{5}{s^2}$$

L_1 and L_2 touch each other, so $\Delta_1 = 1$

$$\frac{x_1}{u} = \frac{\frac{5}{s^2}}{1 - \left(\frac{1}{s} + \frac{10}{s^2}\right)}$$