EECE 5840: HW 1 : A Practical Understanding of Probability, Random Variables and Stochastic Processes by Simulation Due Date: Sept. 17, 2018

Reading Assignment: Chapters 1 and 2 of text book (Kay).

In this first assignment, we will begin by developing a good insight and a practical understanding of the concepts of probability, random variables and stochastic processes. This will require that you be able to write programs using a platform such as Matlab or Python. The following tasks will take you through a systematic development of several of the theoretical concepts we will study using a computation and simulation process.

(1)**Generation of Uniform Random Numbers:** Write a program to generate real valued random numbers that are uniformly distributed between the values (a, b). That is, the random variable x is such that $a \leq x \leq b$. The parameters of the uniform random variable are a and b.

Note that random numbers are computed using what is known as a pseudo-random number generator algorithm. This requires an initial condition that is also known as a seed value. Most computer languages allow you to specify a unique seed value that allows you to generate a controlled and unique sequence of random numbers.

(a) Generate N_E number of ensembles of a set of random numbers by specifying a unique seed value for each ensemble. Let N_x be the number of random numbers or samples per ensemble. The random numbers of the j^{th} ensemble are denoted as $x_j[i], i = 1, 2....N_x, \quad j = 1, 2...N_E$. Note that the index i represents a trial number of the j^{th} experiment. The sequence $x_j[i]$ for a fixed ensemble j is an example of a stochastic or random process, where outcomes may be recorded at successive times denoted by the index i. The ensemble index however represents the repetition of the experiment on another day or time, under similiar operating conditions.

(2)**Computing the statistics of the random numbers**: Next, you will write functions to generate two ensemble averaged statistics of the data you generated in (1): (a) Mean $\mu_X[i]$ and (b) Variance $\sigma_X^2[i]$. The estimate of the mean and variance is given as,

$$\mu_X[i] = \frac{1}{N_E} \sum_{j=1}^{N_E} x_j[i] \tag{1}$$

$$\sigma_X^2[i] = \frac{1}{N_E} \sum_{j=1}^{N_E} x_j^2[i] - \mu_X^2[i] \quad i = 1, 2....N_x \tag{2}$$

Plot $\mu_X[i]$ and $\sigma_X^2[i]$ as a function of i and present your observations for $N_E = 10, 100, 1000$ and $N_x = 50$. Consider two cases: $(a, b) : (0, 1)$ and $(a, b) : (0, 5)$ and interpret the effect of these parameter changes on results of the mean and variance.

(3)**Compute the histogram of the random numbers**: For this problem, consider one representative ensemble of the data and repeat the experiment for $N_x = 50, 100, 1000$. Since the random variables are real numbers, the generation of a histogram will require that you select a bin size that determines the resolution of interest. Apply different bin sizes and examine the shape of the histogram. How does the shape change with change in N_x and with change in the parameters? Provide a brief explanation of your results.